Phase pre-emphasis for PAPR reduction in optical OFDM systems based on OIDFT or time lens

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Phase pre-emphasis is theoretically studied and introduced to reduce peak-to-average power ratio (PAPR) in optical orthogonal frequency division multiplexing (OFDM) systems. It is well treated as a promising candidate for long-haul and high-speed transmission, due to its high spectral efficiency, relatively low bit rate, and advanced robustness against chromatic dispersion and polarization mode dispersion. Optical OFDM has also been applied to wavelength-division multiplexing passive optical network (WDM-PON) and radio-over-fiber (ROF) systems.

High peak-to-average power ratio (PAPR) is a serious intrinsic defect of optical OFDM systems, exacerbating nonlinear impairments in optical fibers and thus deteriorating system performance. Therefore, reduction of PAPR and mitigation of nonlinear impairments have drawn great attention. In wireless OFDM, many PAPR reduction schemes have been proposed and widely studied, including clipping and filtering, partial transmit sequence (PTS), selection mapping (SLM), and so on. Most schemes depend on electrical processing, which needs additional digital signal processing or electrical circuits, and thus increase system complexity and cost. This letter focuses on OFDM systems that implement inverse Fourier transform in optical domain and studies PAPR reduction by optical methods. It should be noticed that SLM is also related with adding different phase rotations to subcarriers. However, the phase pre-emphasis stated in this letter differs from SLM mainly in two aspects: 1) only one combination of phase shifts and one inverse fast Fourier transform (IFFT) operation are needed; 2) optimized phase values are obtained by calculating an equation proposed in this letter.

Phase pre-emphasis means that optimized phase values are pre-chirped to optical pulses on different subcarriers before IFFT in OFDM systems. In this letter, PAPR reduction by phase pre-emphasis scheme is analyzed in two kinds of optical OFDM systems. We propose to use pre-emphasis for PAPR reduction in optical inverse discrete Fourier transformer (OIDFT) based optical OFDM systems, where only one system with 16 subcarriers was studied. Here, a more extensive study is given, with two kinds of optical OFDM systems and different numbers of subcarriers. It is found that phase pre-emphasis is highly effective in PAPR reduction when intensity modulation (IM) is employed, both in time lens-based optical OFDM and in OIDFT based optical OFDM.

An equation is developed to calculate optimized phase pre-emphasis values. Through simulation, the phase values obtained by calculating the equation are proved to be almost as effective as those values optimized by repeated testing. Moreover, optical implementing methods of phase pre-emphasis are proposed and analyzed. Phase pre-emphasis reduces fiber nonlinearity significantly. In a time lens-based OFDM system, it results in a 5.2-dB increase of launch power at the bit error rate (BER) of $10^{-6}$.

Figure 1 depicts a block diagram of an optical OFDM system based on OIDFT. A continuous-wave (CW) laser is fed into a pulse carver and changes into an optical pulse train. A power splitter divides the optical pulse train into $N$ identical trains, which are modulated with $N$ parallel electrical signals coming from a serial-to-parallel processor (not shown in Fig. 1). $N$ is the number of subcarriers. After IM, an OIDFT converts the modulated optical pulses into OFDM symbols. After fiber-link transmission, an optical discrete Fourier transformer (ODFT) rebuilds the transmitted signals on every subcarrier.

In the system shown in Fig. 1, OFDM symbols $x_n$ can be expressed by

$$x_n = \text{IDFT} \left\{ X_k \right\} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \exp \left( \frac{j2\pi n k}{N} \right),$$

$$n = 0, 1, \ldots, N - 1,$$

![Fig. 1. Block diagram of an OIDFT based optical OFDM system. Rx: receiver.](Image)
where $X_k$ denotes the $k$th modulated signal, $x_n$ is a summation of $N$ complex numbers. When $n = 0$, the $N$ complex numbers have a same phase, resulting in a peak value of $|x_n|^2$. PAPR can be expressed by \cite{22}

$$\text{PAPR}_{\text{dB}} = 10\log \left[ \max_{0 \leq n \leq N-1} |x_n|^2 / \text{mean} |x_n|^2 \right]. \tag{2}$$

Because $\max |x_n|^2$ is achieved when the $N$ complex numbers are in phase, PAPR can be reduced if the in-phase condition is broken, which can be realized by introducing different phase pre-emphasis values to $X_k$. With phase pre-emphasis, Eq. (1) will be changed into

$$x_n = \text{IDFT}\{X'_k\} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \exp(j\phi_k) \exp\left(\frac{j2\pi}{N}nk\right), \tag{3}$$

$$X'_k = X_k \exp(j\phi_k), \ k = 0, 1, \ldots, N - 1, \tag{4}$$

where $\phi_k$ is the $k$th phase pre-emphasis. The waveform of OFDM transmission symbols, as well as their PAPR values, will change owing to phase pre-emphasis, which may result in reduced PAPR values. Values of $\phi_k$ need to be optimized in order to obtain a minimum PAPR. From Eq. (2), it is clear that the distribution of $|x_n|^2$ determines PAPR characteristics. Using Eq. (3), $|x_n|^2$ can be written as

$$|x_n|^2 = \frac{1}{N} \left\{ \sum_{k=0}^{N-1} X_k^2 + 2 \sum_{k=0}^{N-1} \sum_{l=k+1}^{N-1} X_l X_k \right\}, \tag{5}$$

$$Q_n = \frac{1}{N} \left\{ \sum_{k=0}^{N-1} X_k \cos \left[ \frac{2\pi}{N} n(l - k) + \phi_l - \phi_k \right] \right\}, \tag{6}$$

$$Q_n = \sum_{l=1}^{N-1} X_0 X_l \cos \left[ \frac{2\pi}{N} nl + \phi_l - \phi_0 \right] + \sum_{l=2}^{N-1} X_1 X_l \cos \left[ \frac{2\pi}{N} n(l - 1) + \phi_l - \phi_1 \right] + \cdots + X_{N-2} X_{N-1} \cos \left[ \frac{2\pi}{N} n + \phi_{N-1} - \phi_{N-2} \right] = X_0 X_1 \cos \left[ \frac{2\pi}{N} n + \phi_1 - \phi_0 \right] + X_0 X_2 \cos \left[ \frac{2\pi}{N} n + \phi_1 - \phi_0 \right] + X_0 X_{N-1} \cos \left[ \frac{2\pi}{N} n + \phi_{N-1} - \phi_0 \right] + X_1 X_2 \cos \left[ \frac{2\pi}{N} n + \phi_2 - \phi_1 \right] + X_1 X_3 \cos \left[ \frac{2\pi}{N} n + \phi_3 - \phi_2 \right] + \cdots + X_{N-2} X_{N-1} \cos \left[ \frac{2\pi}{N} n + \phi_{N-1} - \phi_0 \right] + \cdots + X_{N-2} X_{N-1} \cos \left[ \frac{2\pi}{N} n + \phi_{N-1} - \phi_{N-2} \right]. \tag{7}$$

To reduce PAPR, suitable values of $\phi_k$ should be chosen to weaken $Q_n$ fluctuation. Under the condition of binary IM, $X_k$ is either 0 or 1, $Q_n$ can be seen as a summation of complex numbers. $X_k$ is a random variable, so the number of non-zero complex numbers varies randomly according to the values of $X_k$. Therefore, the variation of $Q_n$, and so PAPR, will be small if all the complex numbers are symmetrically distributed on a circle. To reduce PAPR, an effective method is to choose suitable values of $\phi_k$ to make the complex numbers in the first or second column in Eq. (7) to be symmetrically distributed on a circle. The first column refers to

$$X_0 X_1 \cos \left[ \frac{2\pi}{N} n + \phi_1 - \phi_0 \right], X_1 X_2 \cos \left[ \frac{2\pi}{N} n + \phi_2 - \phi_1 \right], \cdots, X_{N-2} X_{N-1} \cos \left[ \frac{2\pi}{N} n + \phi_{N-1} - \phi_{N-2} \right].$$

It should be mentioned that it is hard to calculate $\phi_k$ when more columns in Eq. (7) are considered. Here, we only consider the first and second columns, because the number of components in every column decreases from the first to the last. Considering the first column in Eq. (7), suitable values of $\phi_k$ follow

$$\left( \phi_{k+1} - \phi_k \right) - \left( \phi_k - \phi_{k-1} \right) = \frac{2\pi}{N - 1}. \tag{8}$$

A solution of Eq. (8) is

$$\phi_k = \frac{k(1 - k)}{N - 1}. \tag{9}$$

Similarly, when considering the second column in Eq. (7), another solution is

$$\phi_k = \left\{ \begin{array}{ll}
\frac{k(2 - k)}{2(N - 2)} & k = 0, 2, 4, \ldots, N - 2 \\
\frac{(k - 1)^2}{2(N - 2)} & k = 1, 3, 5, \ldots, N - 1
\end{array} \right. \tag{10}$$

We calculate the average of Eqs. (9) and (10) to get the values of $\phi_k$

$$\phi_k = \left\{ \begin{array}{ll}
\frac{k(2 - k)}{4(N - 2)} & k = 0, 2, 4, \ldots, N - 2 \\
\frac{(k - 1)^2}{4(N - 2)} & k = 1, 3, 5, \ldots, N - 1
\end{array} \right. \tag{11}$$

To evaluate the effectiveness of this PAPR reduction scheme and the efficiency of Eq. (11), we use a MATLAB package to calculate PAPR based on Fig. 1 and Eqs. (1)–(4) and (11). Optical pulses are generated and each is indicated by 21 points. OFDM symbols are then obtained by OIDFT operation. After that, OFDM symbols are 4-fold sampled for PAPR calculation. 2000 OFDM symbols are calculated in every simulation. We use two methods to obtain optimized phase pre-emphasis values. One is to choose the optimized values from repeated testing. Each testing uses random values of $\phi_k$. Different combinations of $\phi_k$ are used and the corresponding PAPRs are calculated. When the PAPR is the minimum, we get the optimized values of $\phi_k$. Optimized values are selected from 100 tested combinations. The other is to calculate $\phi_k$ by Eq. (11). It is worth mentioning that only in the simulation of Fig. 2 do we use both methods to obtain optimized phase values, while in all other simulations, we only use Eq. (11) to calculate phase values.
Figure 2 depicts the PAPR distribution, where $x$ and $y$-axes represent reference PAPR0 value and the probability when the PAPR is larger than PAPR0, respectively. In this simulation, $N = 16$. The six curves indicate different situations: 1) without phase pre-emphasis, 2) the average performance of phase pre-emphasis by repeated optimization, 3) the optimum performance of phase pre-emphasis by repeated optimization, 4) the performance of phase pre-emphasis based on Eq. (11), 5) SLM with 2 candidates, 6) SLM with 10 candidates, respectively. Compared with no phase pre-emphasis, phase pre-emphasis (curve 4) shows around 3.7-dB PAPR reduction, when the $\lg[\Pr(PAPR>PAPR0)]=–2.5$. Figure 2 also depicts the performance of SLM with 2 or 10 random candidates. In the SLM simulation, after 2 or 10 phase combinations are randomly chosen in order to introduce phase rotations, the minimum PAPR is selected from the 2 or 10 PAPR values which are calculated in parallel[22]. The more PAPR reduction in SLM is brought by multiple IFFT operations, while phase pre-emphasis needs only one. More importantly, curve 4 performs better than curve 2 and is closer to curve 3 than to curve 2. When $\lg[\Pr(PAPR>PAPR0)]=–2.5$, curve 4 is better than curve 2 by 0.8 dB, while worse than curve 3 by 0.4 dB. This means the performance of phase pre-emphasis by Eq. (11) is better than the average performance of phase pre-emphasis by repeated testing and is close to the optimum performance of phase pre-emphasis by repeated testing, which validates the correctness of Eq. (11).

Figure 3 depicts the PAPR probability distribution of OFDM symbols, where the $y$-axis indicates the probability. In this simulation, $N$ equals 64. It can be seen that the probabilities of high PAPRs become much smaller when phase pre-emphases are used, and that the most probable PAPRs are reduced by about 8 dB.

Figure 4 depicts the differences between complementary cumulative distribution functions (CCDFs)[22]. In this simulation, $N$ varies from $2^4$ to $2^9$. When $\lg[\Pr(PAPR>PAPR0)]=–2.5$, PAPR is reduced by 3.7 dB ($N = 16$), 5.8 dB ($N = 32$), 8.0 dB ($N = 64$), 10.7 dB ($N = 128$), 13.3 dB ($N = 256$), and 16.3 dB ($N = 512$), respectively.

Figure 5 depicts the transmission performance, where $x$-axis indicates the launch power of every subcarrier. In this simulation, an OIDFT based OFDM system ($N=16$) is designed, as shown in Fig. 1. Table 1 gives the simulation parameters. The maximum launch power increases by 36.4% (from 1.4 to 2.2 mW), when BER=$10^{-9}$. And the optimum launch power increases by 27.3% (from 1.1 to 1.4 mW).

Figure 6 depicts a SiO$_2$ planar lightwave circuit (PLC) OIDFT[8,21]. For the page layout limitation, we only
show an OIDFT with 4 subcarriers. The OIDFT consists of nine phase shifters, four directional couplers, and three delay lines. To implement phase pre-emphasis, only two more phase shifters (shown in different color) are needed. One only needs to adjust the phase shifting values of the four phase shifters positioned at the left side. This will only bring about limited device or system complexity. Although the additional number of phase shifters will increase with \( N \), the resulted complexity will not be too much, since the number of additional phase shifters is small compared with the number of original phase shifters; moreover, they can be manufactured at the same time when the original phase shifters are produced.

Figure 7 depicts a block diagram of a promising optical OFDM system based on time lens\cite{18,23}. After subcarrier modulation, optical pulses are processed by an inverse Fourier transformer (IFT) block-by-block and are turned into OFDM symbols. After transmission in fiber-links, optical pulses are processed by an inverse Fourier transformer (IFT) block-by-block and are turned into OFDM symbols. After transmission in fiber-links, the IFT and FT in Fig. 7 are continuous, not discrete. An optical OFDM symbol \( (u_{out}^{FT}(t)) \) generated from IFT can be expressed by\cite{18}

\[
u_{out}^{FT}(t) = F^{-1}[\hat{u}_{in}^{FT}(t') \cdot t \to t/(2\pi S_1)]/\sqrt{-i2\pi S_1}, (12)
\]

\[
u(t) = F^{-1}[\hat{u}(t') \cdot t \to t] = \int_{-\infty}^{\infty} \hat{u}(t') \exp(i2\pi t')dt'.(13)
\]

where \( \hat{u}_{in}^{FT}(t') \) indicates the input to the IFT that contains \( N \) modulated (IM) optical pulses; \( u_{out}^{FT}(t) \) indicates the output of the IFT; \( S_1 \) is the accumulated second order dispersion of the first dispersive medium. By introducing phase pre-emphasis, Eq. (12) will be changed into

\[
u_{out}^{FT}(t) = F^{-1}[\hat{u}_{in}^{IFT}(t')]/\sqrt{-i2\pi S_1} = F^{-1}[\hat{u}_{in}^{IFT}(t') \cdot \exp(i\phi(t'))]/\sqrt{-i2\pi S_1}, (14)
\]

\[
\hat{u}_{in}^{IFT}(t') = \hat{u}_{in}^{IFT}(t') \cdot \exp(i\varphi(t')). (15)
\]

In Eq. (15), \( \varphi(t') \) indicates the phase pre-emphasis. By the same method discussed before, we calculate PAPR values using Fig. 7 and Eqs. (12)–(15). Optical pulses are generated and each is indicated by 21 points. OFDM symbols are then obtained after IFT operation. After that, OFDM symbols are 4-fold sampled for PAPR calculation. Also, \( N \) varies during simulations and 2000 OFDM symbols are calculated every time.

Figure 8 depicts the PAPR probability distribution \( (N = 64) \). The curve of PAPR probability distribution shifts noticeably, with a PAPR reduction of around 10 dB. Figure 9 depicts the differences between CCDFs. PAPR reduction is also quite effective in this time lens-based optical OFDM system. When \( \lg[\Pr(\text{PAPR}>\text{PAPR}0)]=-2.5 \), PAPR has been reduced by 4.14 dB \( (N = 16) \), 5.92 dB \( (N = 32) \), 7.64 dB \( (N = 64) \), 10.26 dB \( (N = 128) \), 13.04 dB \( (N = 256) \), and 15.48 dB \( (N = 512) \), respectively.
In the IFT, the quadratic PM is driven by a parabolic voltage generated by an arbitrary waveform generator (AWG)\cite{23}. The parabolic voltage can be replaced by a more practical waveform as shown in Fig. 10(a). In practical implementations, the driving voltage’s amplitude can be confined to $2V_\pi$ \cite{23}, where $V_\pi$ is the half wave voltage of the modulator. Moreover, the driving voltage is periodical, the frequency of which equals $1/N$ of the baud rate \cite{23}. To introduce phase pre-emphasis, one only needs to adjust the driving voltage of the PM. Figure 10(b) depicts the additional driving voltage resulting from phase pre-emphasis ($N = 64$). By adding these two voltages together, we get another periodical driving voltage as shown in Fig. 10(c), whose maximum amplitude and frequency are the same as those of the original driving voltage. Using this summed voltage to drive the PM, both Fourier transform and phase pre-emphasis will be implemented. The implementation of phase pre-emphasis only needs one additional processing: adjusting the driving voltage for PM, which certainly will not result in noticeable device or system complexity. From Eqs. (4) and (15), one can see that phase pre-emphasis will result in no change to the receivers if direct detection (DD) method is employed, for the phase message will be ignored then.

Figure 11 depicts the effectiveness of PAPR reduction clearly, where $x$ and $y$ axes indicate the number of subcarriers and the PAPR reduction, respectively. It is obvious that phase pre-emphasis is more effectual when the number of subcarriers is larger. Performances differ slightly in OIDFT-based OFDM and in time lens-based OFDM. This difference might result from the diversity between discrete Fourier transform and continuous transform.

Figures 12 and 13 depict the transmission performance.

![Fig. 11. PAPR reduction by phase pre-emphasis.](image)

![Fig. 12. BER versus launch power in time lens-based OFDM.](image)

![Fig. 13. Eye diagrams (a) without and (b) with phase pre-emphasis.](image)

In this simulation, a time lens-based optical OFDM system is designed, as shown in Fig. 7. Simulation parameters are given in Table 2. Owing to the reduction of PAPR resulting from phase pre-emphasis, the maximum launch powers are enhanced by 5.2 dB (from 7.6 to 25.5 mW) when BER=$10^{-6}$. And the optimum launch power increases by 36.8% (from 3.8 to 5.2 mW). Phase pre-emphasis also results in an improved eye diagram received after 160-km transmission, as shown in Fig. 13.

In conclusion, phase pre-emphasis is theoretically studied and introduced to reduce PAPR in optical OFDM systems. In IM optical OFDM systems, noticeable PAPR reductions are achieved: 3.7 dB ($N = 16$), 16.3 dB ($N = 512$) in OIDFT based OFDM systems; and 4.14 dB ($N = 16$), 15.48 dB ($N = 512$) in time lens-based OFDM systems. The PAPR reduction results in greatly mitigated nonlinear effects in optical fibers. In an OIDFT based OFDM system, phase pre-emphasis reduces fiber nonlinearity and results in a 36.4% increase of maximum launch power per subcarrier at the BER=$10^{-9}$. In a time lens-based OFDM system, a 5.2-dB increase of launch power is obtained at the BER of $10^{-6}$. An equation is developed to calculate phase pre-emphasis values and is proved to be effective. Moreover, optical implementation methods of phase pre-emphasis are proposed and analyzed. In these two kinds of optical OFDM systems, phase pre-emphasis will only introduce limited device or system complexity. Still, this scheme is ineffective in quadrature amplitude modulation/phase shift keying (QAM/PSK) optical OFDM systems, because phase is used to carry information and is not free for PAPR reduction.

Table 2. Transmission Parameters of Time Lens-Based OFDM

<table>
<thead>
<tr>
<th>Data Rate</th>
<th>160 Gb/s</th>
<th>Modulation Format</th>
<th>IM</th>
</tr>
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<tbody>
<tr>
<td>Transmission Distance</td>
<td>160 km</td>
<td>No. of Subcarriers</td>
<td>1024</td>
</tr>
<tr>
<td>Amplifier Spacing</td>
<td>80 km</td>
<td>Amplifier Noise figure</td>
<td>4 dB</td>
</tr>
<tr>
<td>No. of Bits</td>
<td>$2^{16}$</td>
<td>Fiber Nonlinearity $1.1099$</td>
<td>(G.655) $\cdot$ (km-W)$^{-1}$</td>
</tr>
<tr>
<td>Fiber Loss</td>
<td>0.21 dB/km</td>
<td>Dispersion $(G.655)x$</td>
<td>ps/(km-mm)</td>
</tr>
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</table>

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