Dynamic steering beams for efficient force measurement in optical manipulation

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An efficient and inexpensive method that uses a glass plate mounted onto a motorized rotating stage as a beam-steering device for the generation of dynamic optical traps is reported. Force analysis reveals that there are drag and trapping forces imposed on the bead in the opposite directions, respectively, in a viscous medium. The trapped bead will be rotated following the beam’s motion before it reaches the critical escape velocity when the drag force is equal to the optical trapping force. The equilibrium condition facilitates the experimental measurement of the drag force with potential extensions to the determination of the viscosity of the medium or the refractive index of the bead. The proposed technique can easily be integrated into conventional optical microscopic systems with minimum modifications.

Existing state-of-the-art optical manipulation systems[4−8] use multiple optical beams for multiple parallel manipulations by altering either the temporal[4−6] or spatial parameters[7,8] of the incident beams. This has greatly increased the work capacity of optical manipulation technologies. Furthermore, spinning and orbiting micron-sized particles have found new applications in the microrheology of biological fluids[9] as well as the effective measurement of the quality value Q of an optical trap[10]. With the advancement of optical systems, it would be appealing to incorporate such advancement into optical trapping systems with dynamic manipulation effects. Automated steering techniques, including acousto-optic deflectors (AOD), piezoelectric mirrors, electro-optics, and galvanometers, can be used to demonstrate the measurement of the Stokes drag force, optical manipulation efficiency, time scale of a Brownian motion of a microparticle, and time-sharing multiple optical micromanipulations[4−11].

The use of scanning mirrors mounted on computer-controlled galvanometers[4,5] was one of the first automated beam-steering approaches for optical manipulation. Commercial galvanometers have a low response time of 100 µs and a fast scanning rate of 1−2 kHz. A typical galvanometer laser scanning mirror functions like an analogue galvanometer. With a combination of two such mirrors driven by a galvanometer in an optical manipulation setup, the laser scanning area can cover any position in the transverse plane. A marked improvement from the galvanometer scanning optical trap is the incorporation of piezoelectric systems to automate the beam-steering process[6]. In the same way, a mirror is mounted onto a piezoelectric stage that is controlled by a computer. By applying rapidly alternating electric fields, optical scanning frequencies of up to 10^6 Hz may be achieved. A single piezoelectric scanning mirror may steer the optical manipulation beam in any position on the transverse plane.

Instead of using mechanical means to steer the optical beam, crystal structures altered by electric and acoustic fields can act as a beam-steering device, such as electro-optic deflector (EOD) or acousto-optic deflector modulator (AOD/AOM). An EOD consists of a crystal in which the refractive index can be changed through the application of an external electric field. A change in the refractive index along one plane of the crystal causes a deflection of the input light by an angle, with the switching frequency at around 10^7 Hz. On the other hand, an AOD/AOM uses optical diffraction grating generated by an acoustic wave, driven by a piezoelectric transducer, inside a transparent crystal to steer the laser beam’s transverse direction. A pair of AOD/AOM must be used in order to steer the optical manipulation beam within the transverse plane[5].

Inspired by the steering techniques, we propose and demonstrate a simple, efficient, and inexpensive optical system for force measurement based on the equilibrium of forces in the transverse plane in rotation. In our setup[12−14], as shown in Fig. 1, we use a single tilted glass plate mounted onto a stably rotating optical stage placed behind the conjugate lens pair[15]. The laser used for the optical micromanipulation experiment is a 10-W Yb fiber laser (IPG) at 1064 nm. When the laser beam passes through a tilted glass plate, it undergoes optical refraction that deflects the input beam at an angle of θ, as shown in Fig. 2. By rotating the tilted glass plate around the input beam’s propagating axis using a mechanical rotating stage, a rotating laser beam with a radius of r, proportional to the glass plate’s tilted angle θ, is thus achieved, as described mathematically by

\[
r = t \sin \left(1 - \frac{\cos \theta}{\sqrt{n^2 - \sin^2 \theta}}\right), \tag{1}
\]

which is derived from Snell’s law of refraction and geometrical ray optics. In Eq. (1) r is the rotation radius, t is the thickness of the glass, and θ is the tilting angle.
Fig. 1. Schematic of experimental setup.

Fig. 2. Schematic of beam deflection by glass plate. $\theta$: the tilt angle of the glass plate ($\theta = \phi_i$); $\phi_i$ and $\phi_r$: the angles of incidence and refraction ($\sin \phi_i = n \sin \phi_r$); $n$: the refractive index of the glass plate ($n_{\text{air}} = 1$); $d$: the deflected displacement due to the refraction.

The glass plates are standard-quality microscopic glass plates with high measured transmission of 85–90%. The rotating optical stage, as shown in Fig. 2, is driven by a torque servo drive designed to operate standard three-phase brushless direct current (DC) servo motors (OEM670T Compumotor). It uses a three-state current control for efficient drive performance. The torque and rotating direction are controlled by the analogue voltage input into the command terminal of the controller. In this way, there is no need to design a software interface to control the rotation rate.

When directing the rotating laser beam into a high numerical aperture (NA) objective lens (NA = 1.25, 100×, oil), the diameter of the rotation needs to be less than or equal to the required collimated laser beam diameter, $d$~2 mm, in order to achieve an optimized laser beam focus spot size of around 1.2 $\mu$m, by using $\lambda = 1064$ nm for our experiment[7].

The diameter of the focused spot size of the laser beam $d_0$ is given by

$$d_0 = \frac{2f\lambda}{dn},$$  \hspace{1cm} (2)

where $n$ is the refractive index of oil, $\lambda$ is the wavelength of the laser beam, $f$ is the focal length of the microscopic objective, and $d$ is the required collimated laser beam diameter.

Based on Eq. (1), the plots of the deflected displacement at the glass plate are drawn in Fig. 3 for different thickness of the glass plate. The experimental results indicate that in order to maintain an optimum focal spot size of 1.2 $\mu$m in diameter at the sample plate, the maximum deflection angle and deflection displacement must be maintained at around 50° and 2 mm, respectively, for a 4-mm-thick glass plate. In Fig. 4, we show an optically trapped silica microsphere rotating at the radius of around 16 $\mu$m.

It is mathematically shown in Eq. (3) that the transverse drag force can be increased by increasing the radius of the laser scan, instead of the rotational speed of the mechanical stage:

$$F_{\text{drag}} = 6\pi \eta a \left(\frac{2\pi}{T}\right)^r,$$  \hspace{1cm} (3)

where $\eta$ is the viscosity of the medium, $a$ is the radius of the bead, and $T$ is the mean time it takes the bead to travel in one full circle in the medium (average of the time taken to achieve 20 full rotations), and $r$ is the radius of the rotation circle. Based on Eq. (3), the magnitude of the drag force is directly proportional to the rotation radius while maintaining the same rotating speed. In this experiment, a specific rotation rate, $1/T$, can be measured by using a photodiode placed at the back focal plane of the condenser. By varying the rotating radius $r$ through the adjustment of the angle of tilt of the glass plate, the estimated changes in the drag forces $F_{\text{drag}}$ in a medium of known viscosity can be measured.

The trapping force is given by

$$F_{\text{trap}} = \frac{nQP}{c},$$  \hspace{1cm} (4)

where $Q$ is the optical trapping efficiency that is related to the force on the bead and described in terms of dimensionless parameter, $P$ is the laser power, $n$ is the refractive index of the medium, and $c$ is the light velocity.

In a viscous medium, the rotating bead will reach a critical escape velocity when the drag force is equal to

Fig. 3. Angle of tilt versus the deflection displacement at glass plate for different thicknesses of the glass plate.

Fig. 4. Rotation of a single 5.06-$\mu$m high index particle at a circle with diameter of 16 $\mu$m.
the optical trapping force imposed. When such a velocity is reached, the bead will fall out of the optical trap but still maintain a circular motion at a non-uniform angular velocity. The escape force at the critical escape velocity is approximated as the equilibrium status between the drag force and the trap force, that is,\[
F_{\text{escape}} \approx F_{\text{drag}} = 6\pi \eta a v_{\text{critical}} = 6\pi \eta a \left(\frac{2\pi}{T}\right)r_{\text{critical}}, \quad (5)
\]
and
\[
F_{\text{escape}} \approx \frac{nQ P}{c}. \quad (6)
\]

To measure the critical escape velocity of the bead, we need to control the angular velocity of the mechanical stage to compensate for the changes. In the optical trapping system, both the incident beam power and the radius of the rotation are important parameters to determine the critical escape velocity when the bead and viscosity of the medium are fixed. Figure 5 shows the experimental measurement of the critical escape velocity as a function of the incident power with known viscosity of \(\eta = 1 \times 10^{-3}\) kg/(m-s) of disionized water and silica bead radius of \(r_b = 2.53\) \(\mu\)m for rotation radii \(r\) between 4.5 and 8.4 \(\mu\)m. As can be seen, the critical escape velocities are almost linearly proportional to the power. This experimental result verifies the assumption of the equilibrium conditions in Eqs. (5) and (6) when other parameters are maintained the same. Combining the two equations, we can derive a simple expression of \(Q\) as
\[
Q = \frac{c}{n} \times 6\pi \eta a \times \left(\frac{v_{\text{critical}}}{P}\right). \quad (7)
\]

Equation (7) is also supported by further experimental investigation in multiple sets of measurement of the critical velocity of the silica bead with radius of 2.53 \(\mu\)m for various rotation radii \(r\). In Fig. 6, we plot the measured \(Q\) values based on Eqs. (5) and (6) for rotation radii \(r\) between 4.5 and 8.4 \(\mu\)m. The mean \(Q\) value of the trap is found to fluctuate around 0.1465 for the silica bead with incident power values between 2.78 and 61.24 mW, which correspond to trapping forces of 4.07 and 89.72 pN, respectively. This is consistent with previous calculated measurement[15–19].

In conclusion, we propose and devise a simple, inexpensive, and highly efficient optical steering beam system that can be incorporated into a conventional optical microscope for dynamic optical manipulations with minimum modifications. In addition, we demonstrate and verify the capability of the system for measurement of Stokes drag force with \(Q\) values. The technique can be useful for biological and medical applications when qualitative and quantitative analyses of either the viscosity or the refractive index of biological samples are taken into account.

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References