Equivalence of MTF of a turbid medium and radiative transfer field

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The equivalence of the modulation transfer function (MTF) of a turbid medium and the transmitted radiance from the medium under isotropic diffuse illumination is demonstrated. MTF of a turbid medium can be fully evaluated by numerically solving a radiative transfer problem in a plane parallel medium. MTF for a homogenous single layer turbid medium is investigated as illustration. General features of the MTF in the low and high spatial frequency domains are provided through their dependence on optical thickness, single scattering albedo, asymmetrical factor, and phase function type.

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The spatial frequency \( \Omega \) can be replaced by \( \tan \Omega \) as its unit, the OTF takes \( 1/\text{rad} \) as its unit. Equation (9) or (10) is just the equivalence of MTF of a turbid medium in relation to the transmitted radiance from the medium with isotropic diffuse illumination.

Using this equivalence, full MTF in the whole angular spatial frequency range can be obtained through numerical solution of the radiative transfer problem. Thus, MTF of a single homogenous turbid layer is analyzed for illustration. The radiative transfer problem will be solved by the discrete ordinate method via the DISORT algorithm\(^{[17]} \). A Henyey-Greenstein phase function model \( P_{HG}(\Theta) = (1 - g^2) / (1 + g^2 - 2g \cos \Theta)^{3/2} \) with an adjustable asymmetric factor \( g \) is assigned to the turbid medium, where \( \Theta \) is the scattering angle. The smaller the value of \( g \) is, the more scattered light intensity will be in the side directions. This model has often been used to simulate scattering phase function with a certain degree of scattering asymmetry.

Several general features of MTF are reflected in the following figures. The variations of MTFs for the optical thickness \( \tau \) from \( 10^{-3} \) to 1 are plotted in Fig. 2, and the asymmetric factor \( g \) is set as 0.99. In Fig. 2(a), MTFs are plotted in a wide angular frequency interval and in Fig. 2(b), in a narrower interval (0, 100). For a relatively small optical thickness \( \tau = 10^{-3} \), MTF is close to unity at lower frequency, and decreases with frequency until a critical frequency of approximately 5000 1/rad is achieved. Beyond the critical frequency, the MTF is almost constant. Thus, only the behavior of MTF below the critical frequency is important. The larger the optical thickness is, the lower the critical frequency and the steeper the tendency to descend of the MTF with frequency below the critical value. In the case of the optical thickness being greater than 0.01, the critical frequency will be several ten 1/rad.

\[ J(\vec{\rho}) = \int P(\vec{\rho}' - \vec{\rho}) \, d\vec{\rho}'. \]  
\[ P(\vec{\rho}) = \int \mathcal{O}_{TF}(\vec{v}) \exp \left( 2\pi i \vec{v} \cdot \vec{\rho} \right) \, d\vec{v}. \]

Comparing the two equations above, we obtain directly

\[ \mathcal{O}_{TF}(\vec{v}) = J(\vec{v}'), \]  
\[ \Omega \]

where the variable \( \vec{v} \) takes an identical value of the variable \( \vec{v}' \). Since \( \vec{v} \) and \( \vec{v}' \) are quantities in spatial frequency domain and spatial frequency, respectively, they could take identical values only if they are unit-less. In this letter, it is to our advantage that we are dealing with a problem of imaging at infinite place with an isotropic incident source at infinity. In this case, the absolute positions in both the object and image planes are unnecessary and an angular representation could be used, i.e., \( \vec{\rho} \) can be replaced by \( \tan \theta \), where \( \theta \) is the polar angle of \( \vec{\rho} \), and the spatial frequency \( \vec{v} \) can be replaced by the angular spatial frequency \( \Omega \). Since \( \tan \theta \) is unit-less, the above requirement will be satisfied. Therefore, we have

\[ \mathcal{O}_{TF}(\Omega) = J(\tan \theta). \]  
\[ \Omega \]

In the small angle limit \( \tan \theta \sim \theta \), when \( \theta \) is taken the radian as its unit, the OTF takes 1/rad as its unit. Equation (9) or (10) is just the equivalence of MTF of a turbid medium in relation to the transmitted radiance from the medium with isotropic diffuse illumination.
Fig. 3. MTF of a homogenous turbid medium with different asymmetric factors. Optical thickness is 0.1.

MTFs for four values of asymmetric factor $g$ at the optical thickness $\tau = 0.1$ are plotted in Fig. 3. In Fig. 3(a), the MTFs are plotted in a wide angular frequency interval and in Fig. 3(b), in a narrower interval (0, 100). The effect of the scattering asymmetry produces different behaviors at lower and higher frequencies. Below the critical frequency, the smaller the asymmetric factor is, the steeper the tendency to descend of MTF in relation to frequency. Above the critical frequency, the smaller the asymmetric factor, the larger the value of MTF.

Although these results may provide important features about the MTF of a turbid medium, the application of the equivalence principle and the RTE computational procedure to the scenario directly to obtain the MTF for more complicated turbid media remains a better option. A practical medium may consist of a number of layers, or even has a reflecting bottom, which could be encountered in remote sensing in the atmosphere of the earth. However, the efficiency of the DISORT algorithm facilitates the acquisition of the MTF of any complicated medium particularly if quantitative optical information about the scattering particles are available.

In conclusion, based on the MTF of a turbid medium reported in this letter, and the theoretical solution of the MTF of a turbulent medium obtained previously\cite{18}, the MTFs of random media, both turbulent and turbid are completely solved quantitatively. Application of adaptive optics on turbulence compensation has achieved considerable progress\cite{19,20}. However, no counterpart technology for the turbid media exists. The general MTF feature of the turbid medium may provide useful reference for developing a new kind of technology.

References