Shaping of focal field with controllable amplitude, phase, and polarization

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We propose a method for configuring the distribution of amplitude, phase, and polarization in the focal region of vector beams. The polarization and phase of incident beam is spatially tailored so that it can produce a focal field that has elaborately prescribed shapes. Our work focuses on the design of a special focus structure with two oval rings, wherein a phase gradient and polarization gradient exist in the inner and outer rings, respectively. The incident light yielding the desired focal field is determined based on an iterative scheme involving vectorial diffraction calculations and fast Fourier transforms. Simulations and experiments demonstrate the generation of a focal field with phase and polarization gradients, which may find applications in optical manipulation.

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The focusing of vector beams through high numerical aperture (NA) lenses has been the topic of much research because an elaborately designed focus can find applications in a variety of fields, including optical data storage, microscopy, material processing, near-field optics, micromechanics, and optical tweezers[1–10]. The common procedure for focus shaping is to configure the phase and/or amplitude profiles of an incident beam so that it yields the desired irradiance distribution in the focal volume. Recently, vector beams with space-variant polarization have drawn much attention as they are able to significantly influence the focusing behavior of light[11–14]. We have reported on the use of polarization-only modulation of incident beams to implement focus shaping[15], which demonstrates the functionality of polarization manipulation. On the other hand, it is well known that the tailoring of the wave-front (or phase profile) of light beam is preferred for the focus shaping because, normally, the phase plays a most important role in controlling the propagation behavior of light field; so we proposed to combine the modulation of the polarization and phase to realize more elaborate and versatile structuring of the focal field[16]. An iterative scheme for realizing a special polarization configuration in focus by simultaneous tailoring of both polarization and phase was reported. In this letter, we extend the study to the design of the vectorial focus that has simultaneously and independently modulated amplitude, phase, and polarization distribution of incident field. The specific shape (intensity, phase, and polarization) configurations in the focal field are achieved by optimizing the modulation of both the polarization and phase of the incident beams. The mathematical expressions of the focused fields are derived based on the vector diffraction theory, and numerical simulations and experimental measurements of tightly focused field intensity distributions are further demonstrated. The simulations and optical experiments on vectorial field shaping are performed to validate the feasibility of our method.

Any polarized vector field can be described by the combination of a pair of orthogonal components, which can be represented in circular polarization basis \( \hat{e}_x, \hat{e}_y \) or in linear polarization basis \( \hat{e}_x, \hat{e}_y \). The transformation between the two sets of base vectors are expressed in the following forms:

\[
\hat{e}_x = \frac{1}{\sqrt{2}} (\hat{e}_x + \hat{e}_y), \quad \hat{e}_y = -\frac{i}{\sqrt{2}} (\hat{e}_x - \hat{e}_y),
\]

\[
\hat{e}_x = \frac{1}{\sqrt{2}} (\hat{e}_x + i\hat{e}_y), \quad \hat{e}_y = \frac{1}{\sqrt{2}} (\hat{e}_x - i\hat{e}_y).
\]

Our endeavor starts with an inhomogeneous beam with random distributed amplitude, phase, and polarization across the cross section of beam, described as \( A(x, y) e^{i\phi(x, y)} \left[ \cos \phi(x, y) \hat{e}_x + \sin \phi(x, y) \hat{e}_y \right] \), where \( A(x, y), \phi(x, y) \), and \( \phi(x, y) \) are the amplitude, phase retardation, and polarization angle of the incident light. Our aim is to optimize three parameters of the incident light by using an iterative scheme to create a desired polarization gradient and phase gradient structure in the focal volume. In this study, it is helpful to denote the incident field, \( \hat{E}_0(x, y) = (\hat{E}_x, \hat{E}_y, \hat{E}_z)' \) \((t \) denoting the transverse of matrix and \( E_x = (E_x \pm iE_y)\sqrt{2} \), in the circular basis, as follows:
The three successive rotation operations describe the azimuthal rotation superimposing the $x$-$z$ plane with the meridional plane of the local ray, the refraction through the lens on the angle $\theta$ therein, and the reverse rotation to the global coordinate frame. When applying to the field $(E_x, E_y, E_z)$ in the circular basis, the transformation $\hat{T}$ acquires the following matrix form:

$$\hat{T}(\theta, \phi) = \sqrt{\cos \theta} \begin{pmatrix} a_1 & -a_2 e^{-2i\phi} & \sqrt{2a_1 a_2} e^{-i\phi} \\ -a_2 e^{2i\phi} & a_1 & \sqrt{2a_1 a_2} e^{i\phi} \\ -2a_1 a_2 e^{-i\phi} & -2a_1 a_2 e^{i\phi} & a_1 - a_2 \end{pmatrix},$$

where $a_1 = \cos^2(\theta/2)$ and $a_2 = \sin^2(\theta/2)$.

The focal field of a monochromatic light beam passing through an aplanatic lens is calculated by the vectorial diffraction integral\[\text{[15]}\], which can also be written in terms of a Fourier transform (FT)\[\text{[20, 21]}\]. For the sake of simplicity, we restrict our attention to the paraxial focusing in which the longitudinal component of the focal field can be omitted. If the tight focusing is necessitated, the longitudinal component must be taken into account and the triple components must be computed. On decomposing the circular polarization into a pair of orthogonal basis vectors, the left- and right-hand circular polarization components, the focal field at the point $(x, y, z)$ with respect to the focus can be calculated by\[\text{[16]}\]

$$E_j(x, y, z) = \begin{pmatrix} E_{j1}(x, y, z) \\ E_{j2}(x, y, z) \end{pmatrix} = \int \frac{T}{2f} \left[ A(x_0, y_0) e^{i\alpha(x_0, y_0)} - A(x_0, y_0) e^{i\alpha(x_0, y_0)} \right] \frac{2\pi}{Af} (xx_0 + yy_0) \sin x_0 dy_0,$$

where $(x_0, y_0)$ represents the exit pupil $S$ of the lens, and $\hat{T}_2$ is the two-column counterpart of $\hat{T}$, expressed by

$$\hat{T}_2(\theta, \phi) = \sqrt{\cos \theta} \begin{pmatrix} a_1 & -a_2 e^{-2i\phi} \\ -a_2 e^{2i\phi} & a_1 \end{pmatrix}. \tag{7}$$

Equation (6) indicates that the focal field in a plane at a given axial position is obtained by a two-dimensional FT, which can be implemented efficiently on a computer with the help of algorithms such as the fast FT (FFT).

Our method simulates an iterative process between the forward and backward propagations that relates the incident and focal fields, which is analogous to the Gerchberg–Saxton (GS) algorithm for retrieving the phase of a pair of light distributions related via the FT\[\text{[22]}\]. The use of the GS algorithm in the implementation of our method allows for the fast shaping of the focal field. The flow chart of the iterative algorithm is depicted in Fig. 2. The iterative process starts with assuming a set of randomly distributed amplitude and

![Fig. 1. Geometry of focusing system.](image-url)
phase of the circular components of the incident beam (i.e., \( \alpha \) and \( \beta \)) and the phase factors of the polarization components of the incident light, \( \alpha \) and \( \beta \), are set to the same helical phase of topological charge 1. As elaborated in Refs. [19, 26], the momentum density of optical field can be decomposed into two terms, which are referred to orbital flow density (OFD) and spin flow density (SFD), respectively.

The OFD is calculated by \( \mathbf{p}_o = \text{Im}(\mathbf{E} \cdot \nabla \mathbf{E}) \), the SFD is calculated by \( \mathbf{p}_s = \mathbf{V} \times \left[ \mathbf{i} \cdot (\mathbf{E} \times \mathbf{E}) \right] \). The OFD is associated with phase gradient of the optical field and the SFD is associated with polarization gradient. The direction of OFD is identical with the phase gradient; however, the direction of SFD is orthogonal to the polarization gradient. After attaining the electric field distribution, the SFD and OFD of the focused field can be easily calculated and are plotted in Figs. 3(g) and (h), respectively. As shown in the figure, the OFD and SFD are spatially divided and concentrated in the inner and outer oval stripes, respectively, in which the blue arrowheads indicate the direction of each flow.
Fig. 3. Simulated and experimentally realized focal field with double oval-shape: (a) polarization state of the focal field, intensities of (b) total field, (c) right- and (d) left-circular components, phase structures of (e) right- and (f) left-circular components, and numerically calculated SFD and OFD in the focal field. The length of the arrows on the diagram indicates relative magnitude of the flow density. The intensity of the total field is shown in the background of each frame. (i) CCD recorded intensity of the focal field that is experimentally realized and intensities of the focal field after (j) horizontally and (k) vertically oriented polarizers.
Figure 3(a) indicates that the polarization state varies along the radial direction within the outer oval stripe, which gives rise to the azimuthal SFD confined in this region, as shown in Fig. 3(g). Meanwhile, Figs. 3(e) and (h) indicate that the inner oval stripe possesses a helical phase that is responsible for the azimuthal OFD shown in Fig. 3(h). The experimentally generated focal field is shown in Figs. 3(i)–(k), in which the intensity distribution with no analyzer and after being transmitted through the analyzer (whose orientation is indicated by the arrow) is demonstrated.

In conclusion, we demonstrate the possibility of generating focal fields with the controllable amplitude, phase, and polarization state. Furthermore, the spin and orbital flow in the focused field is manipulated independently. Our method provides a flexible way to tailor the optical field and thus helps us further explore engineering of vector beams targeted for specific applications of a high NA system, for example, in particles trapping and manipulation, which can benefit the investigation of the mechanic effect of vector beams.

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References