Precise alignment of off-axis three-mirror reflecting optical system based on phase diversity

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Phase diversity (PD) is a kind of wavefront sensing technology based on image collecting and post-processing. We apply the PD technology to align an off-axis three-mirror reflecting anastigmatic system precisely. It can be concluded that the wavefront error obtained by PD agrees well with the interferometric result. The focused images are also restored according to the testing results of PD, and the qualities of restored images are improved.

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The PSF can be obtained by performing Fourier transform on the pupil function as

$$\text{PSF}_1 = |F\left\{A(x,y)\exp[i\phi(x,y)]\right\}|^2,$$  
(3)

$$\text{PSF}_2 = |F\left\{A(x,y)\exp[i(\phi(x,y)+\phi_d(x,y))]\right\}|^2,$$  
(4)

and the phase function is expanded on a set of Zernike polynomials. Indeed, aberration in an optical system which can be mathematically represented by Zernike polynomials as

$$\phi(x,y) = \sum_{i=1}^{K} a_i Z_i(\rho, \theta),$$  
(5)

$$\phi_d(x,y) = a_i Z_i(\rho, \theta),$$  
(6)

where $A$ denotes the binary aperture function, $\phi$ is the unknown pupil function, $\phi_d$ is the additional diversity function, $K$ is the number of Zernike polynomials, $Z_i$ is the $i$th Zernike polynomial, and $a_i$ is the $i$th Zernike coefficient. Theoretically, $K$ should tend to infinity to describe any waveform, but in our case of static aberration estimation, the first nine polynomials are enough to describe the aberrations.

According to the maximum-likelihood theory, after some mathematical processes, the final form of merit function is shown as

![Fig. 1. Optical layout of PD.](image-url)
where $\gamma$ is the nonnegative coefficient of regular term, and it can improve the stability and convergence efficiency of the algorithm. When $E$ in Eq. (7) reaches its minimum value, the solved OTF will agree with its actual OTF, and the coefficients of Zernike polynomials can be solved. Thus, the wavefront of system can be fitted. The flow chart of PD algorithm is shown in Fig. 2. According to the aforementioned theory and algorithm, the adaptability analysis of PD technology is accomplished[6].

We have applied PD technology to align an off-axis three-mirror reflecting anastigmatic (TMA) system precisely[7,8]. The experimental setup is shown in Fig. 3.

An integrating sphere is applied as the light source, and a filter centered at 645.32 nm is used. The resolution target is illuminated as an infinite object. The $F$ number ($F#$) of optical system is 9.83. The camera is FL2-20S4M-C charge-coupled device (CCD; Point Grey Corp.), the effective pixel number is 1600×1200 and its single pixel size is 4.4×4.4 ($\mu$m). The CCD camera is located on a 6D adjusting mount. Furthermore, the designed defocus is introduced by moving the CCD along the direction of the incident chief ray and a micrometer is used to monitor the defocus distance as shown in Fig. 4.

The initial position of the CCD is decided in alignment, where the actual defocus W020 of the optical system is practically zero. This initial position can also be regarded as the nominal ideal image plane.

According to the coarse-aligned TMA optical system, two images are collected: one image is in focus and the other is diversity image whose defocus distance is four times the depth of focus, that is, 0.499 mm. The same areas of 128×128 (pixel) of both images are clipped from the original images for simplicity. In order to verify the result of PD technology, the interferometric testing of the system is also carried out and its measured data are set as the criterion. It can be concluded from the comparison as shown in Table 1 and Fig. 5 that the result solved by PD technology agrees well with the interferometric testing.

Fig. 2. Flow chart of PD technology.

$$E(u, a) = \frac{1}{2} \sum_{x \in X} \left[ |l(u)|^2 + |l_x(a)|^2 - \frac{1}{\gamma} \left| \frac{l^* (u) OTF_x (u) - l_x^* (a) OTF_x (u)}{\gamma + |OTF_x (a)|^2 + |OTF_x (a)|^2} \right|^2 \right]$$

(7)

Fig. 4. Schematic of imaging equipment.

Table 1. Comparison of Zernike Coefficients by Two Methods (Coarse-aligned)

<table>
<thead>
<tr>
<th>Zernike Item</th>
<th>PD/λ</th>
<th>Interferometer/λ</th>
<th>Deviation/λ</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-0.0342</td>
<td>-0.0038</td>
<td>-0.0304</td>
</tr>
<tr>
<td>6</td>
<td>0.0274</td>
<td>0.010</td>
<td>0.0174</td>
</tr>
<tr>
<td>7</td>
<td>0.4032</td>
<td>0.4381</td>
<td>-0.0349</td>
</tr>
<tr>
<td>8</td>
<td>-0.3965</td>
<td>-0.4426</td>
<td>0.0461</td>
</tr>
<tr>
<td>9</td>
<td>-0.0171</td>
<td>-0.0313</td>
<td>0.0142</td>
</tr>
<tr>
<td>10</td>
<td>0.4997</td>
<td>0.4504</td>
<td>0.0493</td>
</tr>
<tr>
<td>11</td>
<td>0.4835</td>
<td>0.4655</td>
<td>0.018</td>
</tr>
</tbody>
</table>
According to the testing result of coarse-aligned optical system obtained by PD technology, the TMA system is precisely aligned further. The images of different fields of view are collected, and the wavefront aberration of several fields of view is solved by PD technology. In the same way, the interferometric testing is applied to test the wavefront aberration of the above fields of view. The comparisons of the wavefront maps obtained by two methods are shown in Figs. 7–9.

The RMS values of different fields of view obtained by PD technology are 0.0715λ, 0.0682λ, and 0.0703λ, and the testing errors of PD are all less than 0.02λ. The images are also restored by the Lucy–Richardson filter algorithm based on the results obtained by PD and they are compared with the original focused images (Fig. 10).

It can be seen that both the contrast and the definition of the restored images improve compared
Table 2. RMSE and LS Values of Different Images When System is Precisely Aligned

<table>
<thead>
<tr>
<th>Field of View</th>
<th>Evaluation Function</th>
<th>Focus Image</th>
<th>Restored Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>+1</td>
<td>RMSE</td>
<td>79.34</td>
<td>95.35</td>
</tr>
<tr>
<td></td>
<td>LS</td>
<td>162.47</td>
<td>220.35</td>
</tr>
<tr>
<td>0</td>
<td>RMSE</td>
<td>81.54</td>
<td>99.81</td>
</tr>
<tr>
<td></td>
<td>LS</td>
<td>165.61</td>
<td>228.62</td>
</tr>
<tr>
<td>-1</td>
<td>RMSE</td>
<td>79.13</td>
<td>94.86</td>
</tr>
<tr>
<td></td>
<td>LS</td>
<td>159.07</td>
<td>224.39</td>
</tr>
</tbody>
</table>

with the original images. Also, the RMS error (RMSE) and Laplacian sum (LS) are applied to estimate the quality of the images as shown in Table 2. The definitions of RMSE and LS are shown as

$$\text{RMSE} = \sqrt{\frac{1}{MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} [f(i,j) - f_{\text{mean}}]^2}, \quad (8)$$

$$\text{LS} = \frac{\sum_{i=2}^{M-2} \sum_{j=2}^{N-2} |f(i,j) - \sum_{i=0}^{+1} \sum_{j=0}^{+1} f(i,j)|}{(M-2)(N-2)}. \quad (9)$$

In conclusion, we apply the PD technology to align the TMA optical system precisely. When the system is aligned well, the testing results of all fields of view by PD technology agrees well with the interferometric results. The deviations between the wavefront errors obtained by PD testing and by interferometric testing are all less than $0.02\lambda$ (RMS). The future work includes accelerating the mathematical calculation and the PD technology with the broadband light illumination.

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References