Impact of external optical feedback on the beat notes of dual-polarization fiber lasers

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Fiber optic sensors have been developed for decades and proved successful in various areas, with many advantages. One category of fiber optic sensors is fiber Bragg grating (FBG) sensors\textsuperscript{4}, which gauge measurands by discriminating the wavelength of their reflected spectrum. However, wavelength discrimination in the optical domain is frequently complicated and expensive. To come up with this, dual-frequency fiber grating laser or distributed Bragg reflector (DBR) laser-based sensors are proposed\textsuperscript{2}, which make measurements by frequency discrimination of the beat notes of the lasers in the electrical domain, resulting in compact and cost-effective solutions. Within the past few years, researchers have managed to utilize this probe to detect lateral force\textsuperscript{6}, strain and temperature\textsuperscript{2}, displacement\textsuperscript{2}, hydrostatic pressure\textsuperscript{2}, bending\textsuperscript{2}, twisting\textsuperscript{2}, sound and ultrasound\textsuperscript{9,10}, acceleration\textsuperscript{11}, electric current\textsuperscript{12}, and even magnetic fields\textsuperscript{12}. For such sensors, the performance of dual-frequency fiber lasers is very critical, as it determines the achievable measurement accuracy.

There are several factors involved in determining the performance and limitations of dual-frequency fiber lasers, such as the beat note’s intensity noise, its phase noise, which contains information about the lasing linewidth, and its frequency stability. Because the beat note’s frequency is employed in the measurements, the frequency stability and the phase noise are more relevant in determining the performance. For mono-frequency lasers, external optical feedback has been shown as an effective method for frequency stabilization and phase-noise reduction\textsuperscript{14–17}. The corresponding theoretical basis has been well established\textsuperscript{14–17}. For dual-frequency fiber lasers, the application of external optical feedback and its effects still remain seldom touched. We have recently demonstrated through experiments the suppression of the phase noise of the beat note of a dual-frequency fiber laser by external optical feedback\textsuperscript{20}. However the underlying theoretical basis is left unexplored. In this manuscript, we present a comprehensive theoretical analysis of the impact of external optical feedback on the beat notes of dual-frequency fiber lasers. It shows that external optical feedback can effectively suppress the phase noise of the beat note if some requirements are fulfilled. The polarization of the optical feedback is also important for the stability of the fiber laser and can tune the beat note frequency. A side effect of external optical feedback, as demonstrated in the experiments, is a 7% decrease in the sensitivity of dual-frequency fiber laser-based sensors.

Figure 1 shows the schematic of a dual-frequency fiber laser with a wavelength-matched FBG providing external optical feedback. The fiber laser is formed by a couple of FBGs, FBG\textsubscript{1} and FBG\textsubscript{2}, inscribed on a doped fiber with a cavity length of \(d_{\text{DBR}}\). An external feedback cavity with an effective length of \(d_{\text{ext}}\) is composed of FBG\textsubscript{2} and a third FBG, or FBG\textsubscript{3}. A fictitious reflector can then be assumed.

1shows the schematic of a dual-frequency fiber laser with external optical feedback by an FBG reflector.

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in place of FBG2 and FBG3 with Airy’s formulas to describe its overall transmittance and reflectance. It introduces additional gain and additional phases to the laser cavity, resulting in a new lasing frequency to satisfy the phase-matching condition given by

\[
\omega_3 \tau_{\text{DBR}} = \omega_4 \tau_{\text{DBR}} + \frac{\pi r_3 \sqrt{1 + \alpha^2}}{F_{\text{DBR}}} \sin(\omega_4 \tau_{\text{ext}} + \phi_3 + \tan^{-1} \alpha) \\
= 2m\pi, \quad (m = 0, 1, 2, \ldots),
\]

(1)

where \(\omega_3\) and \(\omega_4\) are, respectively, the optical angular frequency before and after feedback, \(\tau_{\text{DBR}}\) and \(\tau_{\text{ext}}\) stand for the round-trip time in the laser and in the external cavity, respectively, \(a\) represents the linewidth enhancement factor of the laser cavity, and \(\phi_3\) is the additional phase introduced inside the laser cavity by FBG3. \(F_{\text{DBR}} = \frac{\pi r_3}{1 - r_i r_f}\) is the finesse of the DBR laser, and \(r_i\) is the amplitude reflectivity of the \(i\)th FBG. A weak external reflection is assumed and the new lasing frequency with external optical feedback should relate to the original free-running lasing frequency as

\[
\omega_b = \omega_a + \frac{\pi r_3 \sqrt{1 + \alpha^2}}{F_{\text{DBR}} \tau_{\text{DBR}}} \sin(\omega_a \tau_{\text{ext}} + \phi_3 + \tan^{-1} \alpha).
\]

(2)

The polarizations at the two laser frequencies are orthogonal due to the intrinsic birefringence inside the laser cavity. Therefore, Eq. (2) can be rewritten for the two polarization modes as:

\[
\omega_{bx} = \omega_{ax} + \frac{\pi r_3 \sqrt{1 + \alpha^2}}{F_{\text{DBR}} \tau_{\text{DBRx}}} \sin(\omega_{ax} \tau_{\text{ext}} + \phi_3 + \tan^{-1} \alpha),
\]

(3)

\[
\omega_{by} = \omega_{ay} + \frac{\pi r_3 \sqrt{1 + \alpha^2}}{F_{\text{DBR}} \tau_{\text{DBRy}}} \sin(\omega_{ay} \tau_{\text{ext}} + \phi_3 + \tan^{-1} \alpha),
\]

(4)

where the subscripts \(x\) and \(y\) represent the horizontally and the vertically polarized modes, respectively. The beat note frequency is the frequency difference of the two modes; thus, the relation between the beat note frequency before and after the external optical feedback can be expressed as

\[
\Delta \omega_b = \Delta \omega_a + \frac{2 \pi r_3 \sqrt{1 + \alpha^2}}{F_{\text{DBR}} \tau_{\text{DBR}}} \sin \left(\frac{\omega_{ay} \tau_{\text{ext}} - \omega_{ax} \tau_{\text{ext}}}{2}\right) \\
\cos \left(\frac{\omega_{ay} \tau_{\text{ext}} + \omega_{ax} \tau_{\text{ext}}}{2} + \phi_3 + \tan^{-1} \alpha\right).
\]

(5)

Here, two approximations, \(\tau_{\text{DBR}} \approx \tau_{\text{DBRx}} \approx \tau_{\text{DBRy}}\) and \(\phi_3 \approx \phi_{3x} \approx \phi_{3y}\), are employed because the frequency difference between the two polarization modes is very small compared to their large optical frequency, and the reflective index is actually about 5 orders of magnitude larger than the birefringence. We safely assume here \(\omega_{ay}\) is bigger than \(\omega_{ax}\) for convenience. Equation (5) shows that the relative phase difference of the polarizations modes of the original and feedback plays an important role in determining the new beat note frequency after feedback; in other words, the feedback’s polarization impacts the new beat frequency.

If a further approximation \(\tau_{\text{ext}} \approx \tau_{\text{ext}} \approx \tau_{\text{ext}}\) is taken, Eq. (5) is re-written as

\[
\Delta \omega_b = \Delta \omega_a + \frac{2 \pi r_3 \sqrt{1 + \alpha^2}}{F_{\text{DBR}} \tau_{\text{DBR}}} \sin \left(\frac{\Delta \omega_a \tau_{\text{ext}}}{2}\right) \\
\cos \left[\left(\frac{\omega_{ax} + \Delta \omega_a}{2}\right) \tau_{\text{ext}} + \phi_3 + \tan^{-1} \alpha\right],
\]

(6)

which results in

\[
\frac{\partial \Delta \omega_a}{\partial \Delta \omega_a} = 1 + \frac{2 \pi r_3 \sqrt{1 + \alpha^2}}{F_{\text{DBR}} \tau_{\text{DBR}}} \cos(\omega_{ay} \tau_{\text{ext}} + \phi_3 + \tan^{-1} \alpha).
\]

(7)

Equation (7) shows that the relative beat note frequency changes before and after the external optical feedback. With external optical feedback, the beat frequency becomes more resistant to disturbances. Therefore, the corresponding frequency stability is enhanced, and the phase noise is decreased. The noise reduction factor can be enhanced by extending the external feedback time \(\tau_{\text{ext}}\), that is, with a longer feedback fiber length. The factor can also be improved with stronger feedback, \(r_j\), as long as the weak feedback condition is satisfied. The factor is also related with the fiber laser’s characteristics, its length and its finesse. For certain parameters, the optimum noise reduction factor is achieved. This factor \(N_{\text{red}}\) on the decibel scale will be

\[
N_{\text{red}} = 20 \log \left(\frac{\partial \Delta \omega_a}{\partial \Delta \omega_a}\right) = 20 \log \left(1 + \frac{2 \pi r_3 \sqrt{1 + \alpha^2}}{F_{\text{DBR}} \tau_{\text{DBR}}} \cdot \frac{\tau_{\text{ext}}}{\tau_{\text{DBR}}}\right).
\]

(8)

The round-trip time in dual-frequency fiber laser \(\tau_{\text{DBR}}\) is determined by the effective length of the fiber laser, \(L_{\text{eff}}\), which is calculated as in Ref. [22].

To prove the validity of the feedback’s stability, an experimental setup was built according to Scheme A (used in feedback length, strength, and sensitivity investigation) in Fig. 2. The lower left part of the system is a free-running dual-frequency fiber laser composed of FBG1 and FBG2 pumped at 980 nm. Light from the right end (FBG2) of the laser is utilized as feedback, with that end serially connected with a 10:90 coupler, a 60 dB variable optical attenuator (VOA), a polarization controller (PC), a piece of single-mode fiber (SMF), and FBG3. In this configuration, a portion of the feedback light passes the coupler and provides monitoring with an optical spectrum analyzer (OSA, Yokogawa AQ6370C). The attenuator is used to change
the feedback strength of the external cavity. The laser’s output is amplified with an erbium-doped fiber amplifier (EDFA) and then split into two. One is sent to an electronic spectrum analyzer (ESA, Anritsu MS2692A) to obtain the beat note’s frequency spectrum and automatically calculate the phase noise, and the other is delivered to the OSA, or a 10 MHz resolution Brillouin optical spectrum analyzer (BOSA, Aragon BOSA200).

The feedback length is mainly determined by the SMF’s length. As shown in Fig. 3, six different SMF fiber lengths, i.e., 0, 10, 17, 30, 50, and 100 m, are used to investigate the dependence of the beat note’s phase noise spectrum on $L_{\text{ext}}$. In each case, the best feedback strength ($\sim$17 dBm) is tuned. Please note here the initial external cavity length, or pigtail length, with no SMF included, $L_{\text{ext0}} = 4.5$ m, should be added on. The phase noise of the same DBR laser in the free-running condition is given as a contrast. The inset is a free-running dual polarization fiber laser’s spectrum acquired by the BOSA.

From the experiments, it is found that with the increase of feedback fiber length, the phase noise is reduced for as high as 28.89 dB for the low frequency domain and over 10 dB for around 1 MHz. However, it is also observed that after a certain length, the reduction effects are counterbalanced as the high-frequency noise components become more apparent at hundreds of kHz. For feedback fibers longer than 50 m, this noise also becomes a major obstacle in polarization adjustment to stabilize the frequency.

The experimental and theoretical results of the phase noise reduction at 10 kHz are given in Fig. 4, and the experimental data are fitted according to Eq. (8). The theoretical value for coefficient $\frac{\pi^2}{c} \frac{1}{\text{FSR}}$ is 0.2271, and the experimental result is 0.1864, which can be considered as a good support to our theoretical analysis.

Multi-mode oscillations are also observed on the spectra especially with long feedback fibers in our experiments, as shown in Fig. 5. The central frequency drift is possibly induced in the handling and splicing process of different SMFs. It is shown and curve fitted in Fig. 6 that the longer feedback length is, the shorter the mode gap and the higher the possibility of mode hopping will be. This is because the lasing modes’ gap and thus the beat note frequency both are related to the cavity length, as indicated by $\text{FSR} = \frac{2n d_{\text{ext}}}{c}$.

A 30 m long fiber is then incorporated in the feedback system, and different attenuations are induced by the VOA to relate the phase-noise reduction to the feedback strength. The power at the coupler’s output before re-entering the DBR cavity ranges from less than $-120$ dBm (considered a free-running laser) to $-15.5$ dBm, and the phase noise comparison was previously given[23]. The beat’s phase noise spectrum is demonstrated in Fig. 7. It is shown that when...
the feedback is too strong, there will be multi-modes oscillating in the cavity, which also induces noise, as in Ref. [24]. The best feedback strength for this system is around $-17$ dBm.

As shown in Eq. (5), the feedback signal’s polarization also influences the fiber laser’s noise reduction characteristics. To study that further, a system was setup according to Scheme B in Fig. 2. A WDM is used to detour the 980 nm pump light from going into the 50 m long feedback loop and the laser cavity again, the VOA is used to obtain the best feedback strength, and a PC is used to tune the feedback light’s polarization.

The half-wavelength plate of the PC is initially placed vertically at 90°, while the beat frequency $\nu_0$ is 358.5 MHz. Then, the plate is rotated to change the polarization angle in steps of 4° from 70° to 122°, outside of which range the laser is not stable. The output beat note frequency changes dependently, as demonstrated in Fig. 8. The experimental data are curve fitted and show a period of 1/4. This is reasonable because the polarization state repeats after the polarization rotation angle of $\pi$, and the polarization state rotates twice as the half-wavelength plate tunes. It shows that the lasing frequency and noise reduction are closely related to the feedback light’s polarization. In an ideal case, the feedback’s polarization is the same with the lasing light, when the best noise depression is achieved. While they are orthogonal with each other, it will cause unstable working conditions for the laser and consequently the worst noise suppression.

As the dual-frequency fiber laser is an important kind of sensor, it is important to investigate its frequency response over a lateral weight, i.e., its sensitivity, before and after the optical feedback. This experiment is implemented using Scheme A and the probe configuration sketched in Fig. 9; the external fiber length is 30 m with $-17$ dBm feedback. To keep the experiments simple and accurate, the free-running situation is realized with the VOA introducing an over 120 dB round-trip loss, while the feedback case is achieved by tuning it to an appropriate position.

As shown in Fig. 10, the sensitivity for a stabilized dual-frequency fiber laser sensor is found to be 1.4433 MHz/g.
which is about 7.05% less than without feedback, which is 1.6045 MHz/g. The degradation of the sensitivity is shown to be much smaller than the noise suppression level. This is because when sensing is implemented, the fiber laser is subjected to a large disturbance due to the measurand. Although external optical feedback can make the system more stable, such a large disturbance is beyond the capability of the external optical feedback. However, as shown in Eqs. (5) and (6), the operation range is actually periodic in the frequency domain. Therefore, the system will move to a new frequency for stability, which is actually the expected response of sensing. Moreover, the new operation range also moves to and centers at the new frequency, and hence the phase noise is still largely suppressed. Therefore, even though the sensitivity has dropped, it will not likely be as much as the noise level does.

We discuss the phase-noise limit on the beat note frequency of a dual-frequency fiber laser, and then we obtain quantitatively the dependence of the beat frequency stability on the feedback parameter, which is flexibly designable according to our needs. Then, an appropriate working point for this kind of feedback scheme with around 29 dB noise improvement is experimentally realized, and the feedback length, strength, and polarization's relation with the noise is demonstrated and discussed. Along with the phase-noise reduction and frequency stabilization, the optical feedback also brings around a 7% sensitivity drop for this kind of fiber laser sensor.

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