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Reaching a few $10^{-15}$ long-term stability of integrating sphere cold atom clock

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We present the long-term stability of the integrating sphere cold atom clock (ISCAC) and analyze its systematic limitations. The relative frequency instability of $2.6 \times 10^{-15}$ is reached for an averaging time of $2 \times 10^5$ s. The second-order Zeeman effect and the cavity pulling effect in ISCAC, which would induce the frequency drift from the clock transition, are analyzed. The analytical and experimental results indicate that the cavity pulling effect is the main contribution to the long-term frequency instability of the ISCAC. Further technical improvements to the microwave cavity are also discussed.

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Cold atom clocks have undergone rapid development over the last 30 years, since laser cooling was realized[1–3]. The frequency discriminating curves of the frequency standards with cold atom ensembles have a narrower linewidth and smaller Doppler frequency shift than that with a hot vapor cell[4,5]. The integrating sphere cold atom clock (ISCAC)[6,7] and horloge à refroidissement d’atomes en cellule (HORACE)[8] are compact, high-performance cold atom clocks based on diffuse light cooling in a microwave cavity. Its simple design leads to its small volume and low power consumption, which has potential use for space. The short-term frequency stabilities of the ISCAC and HORACE have been widely studied through overcoming some technical problems[9,10]. Nevertheless, there are several systematic effects that can induce frequency shift of the clock transition, mainly including the second-order Zeeman effect, the cavity pulling effect, light shift, collision shift, etc.[11]. The variations of these frequency shifts induce the drift of the clock output, which degrades the long-term performance of the atomic clock[12,13], leading to the rise of the instability of the atomic clock[8].

In this study, the long-term frequency stability of the ISCAC is reported. It is measured by comparing with an H-maser for 10 days, and the frequency flicker floor of $2.6 \times 10^{-15}$ was achieved for an averaging time of $200,000$ s. The physical quantities that induce frequency drift via the second-order Zeeman effect and the cavity pulling effect in ISCAC, which would induce the frequency drift from the clock transition, are analyzed. The analytical and experimental results indicate that the cavity pulling effect is the main contribution to the long-term frequency instability of the ISCAC. Further technical improvements to the microwave cavity are also discussed.

The 6.834 GHz signal is then injected into the microwave cavity for Ramsey interrogating of the cold atoms, which act as the quantum frequency discriminator. The error signal is then fed back to the local oscillator to correct its deviation, generating a high stability and accurate clock signal at 5 MHz. The transition with zero first-order Zeeman shifts between the states of $|F = 1, m_F = 0\rangle$ and $|F = 2, m_F = 0\rangle$ is selected as the clock transition. We place five layers of magnetic shielding to decrease the effect of the geomagnetic field. The quantization axis is provided by a biased field with an amplitude of $\sim 0.6 \mu$T. The vacuum is maintained at $1 \times 10^{-7}$ Pa.

The energy level diagram of the $^{87}$Rb D2 line and the time sequence of the ISCAC are shown in Fig. 2.
where the current is supplied by a commercial low-noise current source (Newport 505B). The output of the current source was measured by a 61/2 digital high-performance multimeter (Agilent 34401 A). The relative current stability is $3.2 \times 10^{-5}$ for an averaging time up to $\tau = 2 \times 10^5$ s, as shown in Fig. 3. Considering the linear relation between the magnitude of the biased field and the coil current, the magnetic field drift is $1.9 \times 10^{-2}$ nT/200.000 s. Thus, the frequency stability of the ISCAC limited by the second-order Zeeman effect is $1.9 \times 10^{-16}$ up to $\tau = 2 \times 10^5$ s.

The cavity pulling effect occurs when the eigenfrequency of the microwave cavity is not tuned exactly to the frequency of the clock transition. The clock frequency shift caused by the cavity pulling effect can be expressed as

$$
\Delta \nu_{cp} = K \frac{\tau_{eff}}{\tau} \left[ \frac{2(\omega_m^2/Q)(\omega_m^2 - \omega_c^2)}{\omega_m^2 - \omega_c^2 + (\omega_m \omega_c/Q)^2} \right] \times \frac{\pi}{4 \tau_{eff}} \int_0^\tau f^2(t) \sin[2\theta(t)]dt,
$$

where

$$
\Delta \nu_z = K_0 |\overline{B}_0| \cdot \hat{z},
$$

$$
\frac{\delta \nu_c}{\nu} = \frac{2K_0 |\overline{B}_0|}{\nu} \delta B,
$$

and

The cooling light, which is 15 MHz red detuned to the resonance of the transition $|F = 2 \rangle \rightarrow |F' = 3 \rangle$, and the repumping light, which is in resonance with the $|F = 1 \rangle \rightarrow |F' = 2 \rangle$ transition, are both injected into the microwave cavity, generating the diffuse light to cool the atoms for 50 ms to $\sim 100 \mu K$[14]. Then, the pumping light, which is in resonance with the transition of $|F = 2 \rangle \rightarrow |F' = 2 \rangle$, transforms the cold atoms in $|F = 2 \rangle$ to three Zeeman sub-levels $|F = 1, m_F = 0, \pm 1 \rangle$. Two $\pi/2$ pulses separated by a free evolution time of 22 ms are used for microwave interrogation. The population of the atoms in the clock state $|F = 2, m_F = 0 \rangle$ is optically detected for 10 ms with the probe beam, which is in resonance with the transition of $|F = 2 \rangle \rightarrow |F' = 3 \rangle$. The clock cycle time is 86.5 ms.

In atomic clocks, several physical effects can induce the drift of the clock signal. The main factors include the second-order Zeeman effect, the cavity pulling effect, light shift, collision shift, etc. Considering the cold atom ensembles with a low density in the cavity and the pulsed operation mode in an ISCAC, the contributions of the collision shift and the light shift to the instability of the atomic clock can be neglected. Here, we will discuss the contributions from the second-order Zeeman effect and the cavity pulling effect.

The frequency shift of the clock transition $|F = 1, m_F = 0 \rangle \rightarrow |F = 2, m_F = 0 \rangle$ for $^{87}$Rb atoms induced by the second-order Zeeman effect can be expressed as

$$
\Delta \nu_z = K_0 |\overline{B}_0| \cdot \hat{z},
$$

where $K_0 = 575.14 \times 10^8$ Hz/T$^2$, $\overline{B}_0$ represents the vector of the applied magnetic field with an amplitude of 0.6 $\mu$T, and $\hat{z}$ is the unit vector of the quantization axis. Therefore, the fractional frequency drift caused by the second-order Zeeman effect can be given as

$$
\frac{\delta \nu_c}{\nu} = \frac{2K_0 |\overline{B}_0|}{\nu} \delta B,
$$

where $\delta \nu_z$ represents the frequency instability owing to the drift of the magnetic field, $\nu_0$ is the frequency of the clock transition, and $\delta B$ represents the fluctuation of the biased magnetic field. Therefore, the magnetic field fluctuation sensitivity of the fractional frequency drift is $1 \times 10^{-5}$ T$^{-1}$ for the second-order Zeeman effect.

The biased magnetic field is generated by a solenoid, where the current is supplied by a commercial low-noise current source (Newport 505B). The output of the current source was measured by a 61/2 digital high-performance multimeter (Agilent 34401 A). The relative current stability is $3.2 \times 10^{-5}$ for an averaging time up to $\tau = 2 \times 10^5$ s, as shown in Fig. 3. Considering the linear relation between the magnitude of the biased field and the coil current, the magnetic field drift is $1.9 \times 10^{-2}$ nT/200.000 s. Thus, the frequency stability of the ISCAC limited by the second-order Zeeman effect is $1.9 \times 10^{-16}$ up to $\tau = 2 \times 10^5$ s.

The cavity pulling effect occurs when the eigenfrequency of the microwave cavity is not tuned exactly to the frequency of the clock transition. The clock frequency shift caused by the cavity pulling effect can be expressed as

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\Delta \nu_{cp} = K \frac{\tau_{eff}}{\tau} \left[ \frac{2(\omega_m^2/Q)(\omega_m^2 - \omega_c^2)}{\omega_m^2 - \omega_c^2 + (\omega_m \omega_c/Q)^2} \right] \times \frac{\pi}{4 \tau_{eff}} \int_0^\tau f^2(t) \sin[2\theta(t)]dt,
$$

where $\Delta \nu_z = \frac{K_0 |\overline{B}_0| \cdot \hat{z}}{\nu_0}$,

$$
\frac{\delta \nu_c}{\nu} = \frac{2K_0 |\overline{B}_0|}{\nu_0} \delta B,
$$

and $\omega_m$ is the frequency of the clock transition. The clock frequency shift caused by the cavity pulling effect can be expressed as

$$
\Delta \nu_{cp} = K \frac{\tau_{eff}}{\tau} \left[ \frac{2(\omega_m^2/Q)(\omega_m^2 - \omega_c^2)}{\omega_m^2 - \omega_c^2 + (\omega_m \omega_c/Q)^2} \right] \times \frac{\pi}{4 \tau_{eff}} \int_0^\tau f^2(t) \sin[2\theta(t)]dt,
$$

where $\Delta \nu_z = \frac{K_0 |\overline{B}_0| \cdot \hat{z}}{\nu_0}$,

$$
\frac{\delta \nu_c}{\nu} = \frac{2K_0 |\overline{B}_0|}{\nu_0} \delta B,
$$

and $\omega_m$ is the frequency of the clock transition. The clock frequency shift caused by the cavity pulling effect can be expressed as

$$
\Delta \nu_{cp} = K \frac{\tau_{eff}}{\tau} \left[ \frac{2(\omega_m^2/Q)(\omega_m^2 - \omega_c^2)}{\omega_m^2 - \omega_c^2 + (\omega_m \omega_c/Q)^2} \right] \times \frac{\pi}{4 \tau_{eff}} \int_0^\tau f^2(t) \sin[2\theta(t)]dt.
$$

Fig. 2. (a) Energy level diagram of the $^{87}$Rb D$_2$ line under the biased field. The dashed lines in the $|F = 1 \rangle$ and $|F = 2 \rangle$ states indicate the unperturbed energy levels. (b) Time sequence of the ISCAC.

Fig. 3. Relative current stability of the solenoid. It is $3.2 \times 10^{-5}$ for an averaging time up to $\tau = 2 \times 10^5$ s.
where \( K = \mu \alpha_\nu \frac{N_{at} Q}{2\pi^2 h V_{\text{mode}}} \), \( \tau_{\text{eff}} = \int_0^T \sin(\frac{\pi t}{\tau}) dt \), \( T_{\text{eff}} = \int_0^{T+2\tau} g(t) dt \), \( \theta(t) = \frac{1}{\tau_{\text{eff}}} \int_0^\tau \sin(\frac{\pi t}{\tau_{\text{eff}}}) dt \); \( a_\nu \) and \( a_v \) represent the frequency of the clock transition and the eigenfrequency of the microwave cavity, respectively; \( Q \) and \( V_{\text{mode}} \) are the quality factor and the mode volume of the microwave cavity, respectively; and \( N_{at} \) is the cold atom number in the clock state in the cavity, which can be estimated through the spatial density distribution of cold atoms in the microwave cavity\(^{16} \). The last factor behind the multiplication sign in Eq. (3) is a dimensionless factor equal to 0.442 for a TE_{011} cylindrical mode with a \( \pi/2 \) Ramsey pulse. \( g(t) \) represents the sensitivity function defined as the response of the transition probability to the frequency fluctuation in the interrogation field\(^{17} \). From Eq. (3), the main factors that induce the frequency shift are the cavity detuning, which is mainly caused by the temperature drift of the microwave cavity, and the cold atom number fluctuation.

Figure 4 shows the simulated results for the fractional frequency shift induced by the cavity pulling effect. The eigenfrequency of the cavity is tuned to be near the clock transition by adjusting the temperature of the microwave cavity. The relative frequency shift with temperature sensitivity of \( 4.2 \times 10^{-14} \text{K}^{-1} \) can be deduced, since the temperature coefficient of the copper cavity is \( 113.5 \text{kHz/K} \).

The temperature of the microwave cavity is recorded by measuring the resistor of the negative temperature coefficient (NTC) thermistor on the cavity. We apply active temperature control by twisted pairs outside the vacuum enclosure to decrease the temperature drift. Figure 5 shows the temperature stability of the microwave cavity over 10 days. The temperature drift is about 60 mK/200,000 s under an environmental temperature of 303 K.

According to the temperature sensitivity of the clock frequency shift induced by the cavity pulling effect, the fractional frequency instability of the ISCAC is limited to \( 2.5 \times 10^{-15} \) up to \( \tau = 2 \times 10^5 \text{s} \). The number of cold atoms \( N_{at} \) mainly depends on the power of the cooling light, which is actively controlled by adjusting the radio-frequency power loading on the acousto-optic modulator\(^{18} \). The relative power stability of cooling light is shown in Fig. 6. It can be seen that the relative power stability is about \( 2.4 \times 10^{-3} \) for an averaging time of 200,000 s, which results in the cold atom number fluctuation of \( 1.7 \times 10^6 \) at 200,000 s. Thus, the clock frequency drift induced by the atom number fluctuation can be deduced to be below \( 1 \times 10^{-16} \) with a fixed frequency detuning of the cavity within \( \pm 30 \text{kHz} \).

The frequency stability of the ISCAC is measured by comparing with an H-maser (iMaser3000), whose frequency stability is about \( 1.2 \times 10^{-15} \) for an averaging time
of 200,000 s, and the long-term frequency drift is $2 \times 10^{-16}$ per day. The short-term frequency stability of the ISCAC goes down to $5.2 \times 10^{-13} \tau^{-1/2}$ for an averaging time between 4 and 10,000 s, as shown in Fig. 7. The limitation factors of the short-term frequency stability of the ISCAC include atomic noise (the quantum projection noise and atomic shot noise), the Dick effect, and technical noise (laser noise and electrical noise)\textsuperscript{6,7}. The flicker floor of the frequency instability reaches $2.6 \times 10^{-15}$ for an averaging time of $2 \times 10^5$ s, which is mainly limited by the cavity pulling effect. In HORACE, the cavity pulling effect is also one factor limiting its long-term performance\textsuperscript{25}.

From the analytical and measured results reported above, it turns out that it is necessary to decrease the cavity pulling effect for improving the long-term performance of the ISCAC. Figure 8 shows the simulated results for the fractional frequency shift as a function of cavity detuning with different quality factors of the microwave cavity and their contributions to the long-term frequency instability of the ISCAC. It can be seen that the temperature sensitivity of the fractional frequency shift can be reduced by decreasing the quality factor of the microwave cavity. The frequency instability can be decreased to $1.9 \times 10^{-16}$ with the quality factor of 3,000, which is the state-of-the-art performance of the compact microwave clock. Nevertheless, the uniformity of the microwave field in the cavity will be degraded with decreasing the quality factor of the cavity, which would decrease the signal-to-noise ratio of the Ramsey fringes and increase the frequency shift from the distributed cavity phase (DCP). Thus, it is important to choose an appropriate value of the quality factor to optimize the performance of the ISCAC.

In conclusion, we presented the long-term frequency stability of $2.6 \times 10^{-15}$ for an averaging time up to $2 \times 10^5$ s of the ISCAC, whose frequency flicker floor was achieved. The measured data are in good agreement with the analytical results, indicating that the main contribution to the long-term instability of the ISCAC comes from the cavity pulling effect. The quality factor of the microwave cavity should be further investigated to decrease the cavity pulling effect and simultaneously obtain a high uniformity of the microwave field in the cavity for the purpose of reaching the frequency instability of a few $1 \times 10^{-16}$ level of the ISCAC.

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References