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All-dielectric three-element transmissive Huygens’ metasurface performing anomalous refraction

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Metasurfaces have pioneered a new avenue for advanced wave-front engineering. Among the various types of metasurfaces, Huygens’ metasurfaces are thought to be a novel paradigm for flat optical devices. Enabled by spectrally overlapped electric resonance and magnetic resonance, Huygens’ metasurfaces are imparted with high transmission and full phase coverage of $2\pi$, which makes them capable of realizing high-efficiency wave-front control. However, a defect of Huygens’ metasurfaces is that their phase profiles and transmissive responses are often sensitive to the interaction of neighboring Huygens’ elements. Consequently, the original assigned phase distribution can be distorted. In this work, we present our design strategy of transmissive Huygens’ metasurfaces performing anomalous refraction. We illustrate the investigation of Huygens’ elements, realizing the overlapping between an electric dipole and magnetic dipole resonance based on cross-shaped structures. We find that the traditional discrete equidistant-phase design method is not enough to realize a transmissive Huygens’ surface due to the interaction between neighboring Huygens’ elements. Therefore, we introduce an extra optimization process on the element spacing to palliate the phase distortion resulting from the element interaction. Based on this method, we successfully design unequally spaced three-element transmissive metasurfaces exhibiting anomalous refraction effect. The anomalous refractive angle of the designed Huygens’ metasurface is $30^\circ$, which exceeds the angles of most present transmissive Huygens’ metasurfaces. A transmissive efficiency of 83.5% is numerically derived at the operating wavelength. The far-field electric distribution shows that about 93% of transmissive light is directed along the $30^\circ$ refractive direction. The deflection angle can be tuned by adjusting the number of Huygens’ elements in one metasurface unit cell. The design strategies used in this paper can be inspiring for other functional Huygens’ metasurface schemes.

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1. INTRODUCTION

The shaping and engineering of electromagnetic wavefronts at subwavelength scale have attracted much attention due to their extensive applications in recent years. The control of propagating wavefront enables functional optical devices such as lens, anomalous reflectors or refractors, vortex beam exciters, cloaking, and polarization convertors to name a few [1–8]. These applications are designed to be imparted with particular phase profiles to realize abundant functions. In optical scale, phase engineering can be achieved by the imposition of discrete phase changes on a wavefront using arranged micro- or nano-antenna arrays. Each element inside these antenna arrays acts as an individual phase convertor and changes an incident phase response into a required reflected or transmitted phase response, utilizing their intrinsic electromagnetic resonances or optical modes.

According to the literature, metasurfaces can be roughly divided into two groups, namely plasmonic and dielectric metasurfaces. Plasmonic metasurfaces are plagued by huge intrinsic ohmic loss [9–11], which leads to a decrease in their working efficiency and prohibits their application in high-efficiency systems. Dielectric metasurfaces, however, are characterized
by very low ohmic loss and are preferred in high-efficiency systems. Generally speaking, there are three groups of dielectric metasurfaces. The first group employs Fabry–Perot elements [12–15]. These elements can be regarded as resonating Fabry–Perot cavities, where light reflects back and forth at both ends of these structures. For the Fabry–Perot type metasurfaces, their elements are often required to have enough height to support Fabry–Perot resonances. As a result, they often possess large aspect ratios which make them relatively difficult to produce at nano-scale. The second group of metasurfaces often utilizes circularly polarized incident-light illuminated resonant elements to realize Pancharatnam–Berry phase shift [16–19]. These types of metasurfaces often have high efficiency [20], while the incident light is often limited only to circularly polarized light. The last group of metasurfaces, and the one discussed in this paper, is Huygens’ metasurfaces. Huygens’ metasurfaces adopt the spectral and spatial overlapping between an electric dipole (ED) and a magnetic dipole (MD) resonance to achieve phase change [21–26,27]. The accumulation of $\pi$ phase shift resulting from ED and MD resonance together leads to a $2\pi$ phase shift at the ED–MD overlapping wavelength. According to Kerker effect, the overlapping between the ED and MD resonance results in forward-scattering performance [28,29]. Consequently, the transmission of the Huygens’ elements is enhanced to near unity. Compared with the other two types of metasurfaces, Huygens’ metasurfaces often have low aspect ratios and can be designed to work in any incident conditions, which can compensate for the shortages of the other two types of metasurfaces.

However, Huygens’ metasurfaces are plagued by other problems. For example, the introduction of substrates to the Huygens’ elements leads to a variation in MD and ED resonant radiative patterns [30]. As a result, the Kerker condition is no longer satisfied, and the spectral phase response and transmittance of Huygens’ elements will change. Therefore, some publications have designed substrate-free Huygens’ metasurfaces [31,32], which, however, are hard to apply to real optical systems. The interaction between Huygens’ elements can be another crucial obstacle to the realization of functional metasurfaces. It is well known that Mie resonances arise from induced polarized charges and circular currents. The interaction between neighboring Huygens’ elements will no doubt change the distributions of the induced charges and currents. Consequently, the phase response and transmission of Huygens’ elements inside a designed metasurface can be different from those of isolated Huygens’ elements. Therefore, some publications only introduce how to realize ED–MD spectral overlapping or the design of isolated Huygens’ elements, but they do not realize the design of functional Huygens’ metasurfaces [21,22,26]. Some other publications adopt large element spacing to decrease the influence of element interaction or use a large number of Huygens’ elements to maintain phase continuity. However, their functionalities or efficiency is hence limited due to these arrangements. For example, the anomalous refractive angles of present Huygens’ metasurfaces are often less than 20° [23–25], which greatly limits their real applications.

In this paper, we aim at both the design of Huygens’ elements endowed with spectrally overlapped ED and MD resonance, and the realization of efficient transmissive Huygens’ metasurfaces performing anomalous refraction. We will start with the analysis of the ED and MD resonance inside dielectric cross-shaped particles to derive Huygens’ elements performing full $2\pi$ phase response. We will discuss the spectral reflection response and the phase response of these elements, and identify the corresponding elements’ parameters required for full phase coverage. Then we take a step further and discuss the design of anomalous refracting Huygens’ metasurfaces. Considering that the inevitable interaction between these elements can have an undesired impact on the phase and refractive response of the metasurface, we will make an improvement on the traditional Huygens’ metasurface design process and realize functional Huygens’ metasurface design. In the end, we will present that our method is also efficient in designing metasurfaces with more elements, through which the anomalous refractive angles of the metasurfaces can be controlled.

2. DESIGN STRATEGY

A. ED and MD inside an Isolated Cross-Shaped Particle

For a single element of Huygens’ metasurfaces, one of the major requisites is at least one structural variable closely related with the ED and MD resonances inside the structure. Specifically speaking, the ED and MD resonant wavelengths should spectrally overlap with the variation of a certain structural parameter. Some regular-shaped structures, such as spheres and cubes, cannot be applied to the Huygens’ elements because the ED and MD resonances inside these structures are both directly proportional to the size parameters and cannot overlap without any other manipulating processes. The basic element structure chosen for metasurface design in this paper is depicted in Fig. 1(a). Instead of commonly used cuboid- or cylinder-shaped structures, the cross-shaped structure is applied as the fundamental element of our metasurface. Compared with other structures, the cross-shaped structure has certain particular characteristics. At normal TM incidence, its two orthogonal rectangle arms direct towards the incident electric and magnetic fields, respectively. The induced electric dipole moment is along the $x$ direction while the induced magnetic dipole moment is along the $y$ direction, as shown in Fig. 1(c). The orthogonality of the two arms ensures relative independent variation for the ED and MD moments. The variation of one arm of the cross-shaped structure will lead to a stronger influence for one dipole moment and a weaker influence for the other. The inconsistent influence is very convenient for designing ED–MD overlapping elements.

For an isolated particle, the ED and MD scattering cross sections (SCSs) can be used to present the ED and MD resonant wavelengths and strengths. For the cross-shaped structures, the commonly known Mie theory is no longer efficient in acquiring the desired scattering cross sections. Here, we use the electromagnetic multipole expansion (EME) method to calculate the corresponding dipolar coefficients [33]. Electric and magnetic multipolar coefficients inside an arbitrary particle can be expressed as
where the associated Legendre polynomials.

The ordinate origin is set as the center of the cross particle for (a) and (b), while it is set as the center of the cross particle in (d). The refractive index of the substrate is set as 1.45, while the refractive index of the coating layer is set as 1.4. The refractive index of the silicon is set as 3.5. The coordinate origin is set as the center of the cross particle for (a) and (b), while it is set as the center of the cross particle C2 for the structure in (d).

\[
a_E(l, m) = \left(\frac{1}{2\pi}\right)^{1/2} \frac{(-j)^{(l+1)}k^2\eta O_{lm}}{E_0[\pi(2l+1)]^{1/2}} \int \exp(-im\phi) \times \left\{ \frac{\Psi_j^l(kr) + \Psi_{j+1}^l(kr)}{kr} \cdot P_{lm}^m(\cos \theta) \hat{r} \cdot J_{S,j}(r) \right. \\
+ \frac{\Psi_j^l(kr)}{kr}[\tau_{lm}(\theta)\hat{\phi} \cdot J_{S,j}(r) - i\pi_{lm}(\theta)\hat{\psi} \cdot J_{S,j}(r)] \right\} d^3r \times a_M(l, m) = \left(\frac{1}{2\pi}\right)^{1/2} \frac{(-j)^{(l+1)}k^2\eta O_{lm}}{E_0[\pi(2l+1)]^{1/2}} \int \exp(-im\phi) j_j(k, r) \times [\tau_{lm}(\theta)\hat{\phi} \cdot J_{S,j}(r) + i\pi_{lm}(\theta)\hat{\psi} \cdot J_{S,j}(r)] d^3r, \tag{1}
\]

where \(l\) represents the multipole order; \(m\) is an integer; \(\Psi_j^l(kr)\) and \(\Psi_{j+1}^l(kr)\) are the first and second derivatives of the Riccati–Bessel functions \(\Psi_j^l(kr) = krj_j(kr)\) with respect to the argument \(kr\). \(O_{lm}, \pi_{lm},\) and \(\tau_{lm}\) are parameters \([33,34]\); and \(P_{lm}^m\) are the associated Legendre polynomials. \(J_{S,j}(r)\) is the effective current density that creates the scattered field of the \(j\)-th particle in the self-consistent solution of Maxwell equations. The integrals in Eq. (1) are operated only inside the particle. Out of the particle, integrands are equal to zero. The current density distributions and the integrals in Eq. (1) are calculated by the numerical simulation software, COMSOL Multiphysics. As for simulation, the particles are placed in the air, out of which it is a thick perfectly matched layer (PML). The boundary between air and PML is set as scattering boundary condition (SBC). The mesh is manually set with a minimum mesh size of 5 nm. The incident field is set as a normal plane field. The operating condition is set as a built-in scattering-field environment, where the incident electric field can be omitted from the total field. The scattering cross section for a particle can be derived through \([33]\)

\[
C_i = \frac{\pi}{k^2} \sum_{l=1}^{\infty} \sum_{m=-l}^{l} (2l+1)(|a_E(l, m)|^2 + |a_M(l, m)|^2). \tag{2}
\]

The terms of the series in Eq. (2) allow one to determine the contribution of each multipole excitation to the overall SCS. For example, scattering contribution of ED and MD can be calculated through

\[
\sigma_{ED} = \frac{3\pi}{k^2} \sum_{l=1}^{1} |a_E(1, m)|^2, \tag{3}
\]

\[
\sigma_{MD} = \frac{3\pi}{k^2} \sum_{l=1}^{\infty} |a_M(1, m)|^2,
\]

namely when \(l = 1\) for Eq. (2).

In Fig. 2(a), we plot the calculated scattering cross sections of ED and MD components inside isolated cross-shaped structures as a function of the \(x\) arm length \(l_x\). The intensities of the ED and MD resonances both increase with the increment of \(l_x\), while their resonant wavelengths both redshift with the increment of \(l_x\). Such effect is known as the size effect \([35]\). The intensity of the ED resonance increases more prominently than the intensity of MD resonance when \(l_x\) increases. In Fig. 2(b), we plot the ED and MD resonant wavelengths as a function of \(l_x\). Apparently, the ED resonant wavelength redshifts relatively quicker than the MD resonant wavelength, resulting in spectral approaching between the ED and MD resonance with the increment of \(l_x\), whereby the spectral spacing between ED and MD reduces from 200 to about 100 nm with the variation of the parameter \(l_x\). This result proves that ED resonance is more active to the change of the \(x\) arm length \(l_x\), compared with
MD resonance. Such effect can be further applied to realize ED–MD overlapping.

B. ED and MD inside Periodically Arranged Cross-Shaped Structures

In periodically arranged array, ED and MD resonances exist in the form of coupled dipole resonances called the array effect. When the particle size is far smaller than the incident wavelength, the particle array can be regarded as a dipolar moment array with efficient polarizabilities. The efficient polarizability can be calculated as [36]

\[ \frac{\varepsilon_0 / \alpha_{\text{eff}}^{\text{ED}}}{\varepsilon_0 / \alpha_{\text{eff}}^{\text{MD}}} = \frac{k_i^2 G_{xx}^0}{2}, \]

\[ \frac{1/\alpha_{\text{MD}}^{\text{ED}}}{1/\alpha_{\text{MD}}^{\text{MD}}} = \frac{k_i^2 G_{yy}^0}{2}, \]  

Equation (4) also indicates that a single dipole resonance in array, and are mainly determined by the lattice constants in the periodical element array brings about a reflection peak, while the overlapping between an ED and MD resonance leads to a transmission peak. However, elements inside a real meta-surface cannot all be designed to be in ED–MD overlapping state, considering that various phase responses are required. Therefore, some elements are working at the quasi-ED–MD overlapping state or single dipolar resonant state. As a result, the efficiency of a transmissive Huygens’ metasurface is hard to reach unity.

For applicable metasurfaces, substrates are often required to support periodical elements and keep them flush with surrounding elements. However, the introduction of substrates will result in the distortion of dipolar resonant radiative patterns [30]. The original ED or MD polarizability \( \alpha_{\text{eff}}^{\text{ED}} \) or \( \alpha_{\text{eff}}^{\text{MD}} \) will change, and the overlapping condition \( \alpha_{\text{eff}}^{\text{ED}} = \alpha_{\text{eff}}^{\text{MD}} \) will no longer be satisfied. Consequently, the direction or quasi-directional scattering performances vanish, and the transmissive efficiency decreases. To eliminate the influence of the substrate as much as possible, the particles above the substrate can be coated with another dielectric layer, with a close refractive index to the substrate. The sketch of the particle attached with a substrate and coating layer is plotted in Fig. 1(b). The refractive index of the substrate is set as 1.45, while the refractive index of the coating layer is set as 1.4. With this process, the particle can be regarded as being embedded inside a homogeneous layer. Such arrangement only changes the background environment of a particle, namely only the wave vector \( k_i \) changes in Eq. (4). Consequently, the ED and MD resonances inside the array both spectrally shift while their radiative pattern changes slightly, and direction scattering can still be realized. Other publications [21,24] also adopt this arrangement, but the implied physics mechanism is not demonstrated.

In Figs. 3(a) and 3(b), the spectral reflection and phase response of the cross-shaped structure imposed with the substrate and coating layer in Fig. 1(b) are plotted as a function of the arm length \( l_x \). As mentioned before, the ED and MD resonances induce reflection peaks in periodical particle arrays [36,39]. Therefore, we use spectral reflection responses here to address the ED and MD resonant spectral wavelengths inside the periodical structure. As for the simulation of the transmission and phase responses of periodical arrays, we use the software Lumerical FDTD here to derive the required data. Periodical boundary conditions (PBCs) are adopted at both the \( \hat{x} \) and \( \hat{y} \) boundaries of the simulated structures. Perfectly matched layers are imposed on the top and bottom of the simulated unit cell. A minimum mesh size of 5 nm is applied, ensuring that the calculation is independent of the mesh size.

In Fig. 3(a), there are two distinguishable resonant peaks, which are spectrally close to the resonant peaks of ED and MD.
MD in Fig. 2(b). To prove these resonant peaks to be Mie resonances, we plot the corresponding electric field distributions in Fig. 4. The electric vector distributions in Figs. 4(c) and 4(d) confirm the resonant modes to be ED and MD resonances. Referring back to Fig. 3(a), the ED and MD resonances both spectrally redshift with the increment of the parameter $l_x$. However, the ED resonance shows a more active spectral shifting effect than the MD counterpart. Consequently, the two resonant peaks merge near 1.49 $\mu$m. According to the Kerker effect, the spectral overlapping between ED and MD resonance leads to directional forward scattering. Therefore, the two reflection peaks change into a transmission peak at the ED–MD overlapping wavelength in Fig. 3(a). In Fig. 3(b), the phase of the transmitted light is plotted as a function of the structural parameter $l_x$. According to the numerical results, there is a $\pi$ phase-shifting at both ED and MD resonances. When the two resonances spectrally overlap, the total phase shifting of the transmitting light changes to $2\pi$. Applying the low reflection and the swift phase change at around 1.5 $\mu$m in Figs. 3(a) and 3(b), we can further design the desired metasurfaces.

C. Metasurface Design

In general, anomalous refracting angle decreases with increment of the elements inside a periodical unit cell of a metasurface. To derive a large reflecting angle, we set the number of elements inside one unit cell as 3. Traditional Huygens’ metasurface design requires discrete equidistant-phase Huygens’ elements to realize certain phase profiles. Therefore, three cross-shaped structures with an equidistant phase discretization are selected to form the metasurface. The variables of the three cross-shaped structures and the spectral responses are listed in Table 1. The three elements are designed to together realize a $2\pi$ phase shifting while maintaining a relatively high transmission (>87%). The total periodicity of the metasurface is set as $P_{3x} = 3.0136\mu$m, which is three times the periodicity of a single element (1 $\mu$m). The phase and transmission are measured at a selected wavelength $\lambda = 1.507\mu$m. With these arrangements, the metasurface is supposed to realize a linear phase profile at around 1.507 $\mu$m. In Fig. 1(d), we plot the sketch of the designed metasurface. The parametric configuration here is marked as configuration 1.

In Fig. 5(e), we plot the phase distribution of the designed metasurface at 1.493 $\mu$m, where the phase profile is closest to the prespecified linear phase profile. Apparently, the phase of the transmitted light distributes irregularly along the $x$ direction. The real phase response of each element inside the unit cell for configuration 1 is calculated and listed in Table 1. The real phase responses of these elements in the metasurface unit cell turn out to be quite different from the phase responses of these elements in their corresponding periodical arrays. This result can be attributed to the interaction between these elements. In Figs. 5(a)–5(d), we respectively plot the electric field distributions of the three periodically arranged elements, together with the electric field distribution of the metasurface unit cell. Apparently, the interaction between the three particles changes the original field distribution. Consequently, the phase response and transmission of the metasurface will both get changed. Instead of a linear phase profile, the phase profile in Fig. 5(e) shows an irregular change. The transmissions of the original three elements are 87%, 97%, and 97%, respectively.
In most metasurface design schemes, the geometries of the elements, and the periodicity of the unit cell, are determined by the distance between neighboring elements, the metasurfaces. Other methods need to be utilized to realize the desired phase profile through element spacing. With this hypothesis, we then consider the interaction between elements. Instead, the variation of the spacing between elements may have an unexpected influence on the phase profile. With this hypothesis, we then consider how to realize specific phase profiles through element spacing control. However, the interaction between the three non-simple elements is complex, and there is no analytical theory illustrating how to adjust these parameters on purpose.

Therefore, we use a global optimization method here to investigate our assumption.

To reduce the complexity of calculation, some main parameters in Fig. 6(d) are chosen to be the optimizing parameters. The periodicity is fixed as \( P_x = 3 \) and \( P_y = 1 \) \( \mu m \). Of particular importance is that the \( y \)-arm parameters \( \mu_x \) and \( \mu_y \) are fixed during optimization, since these two parameters seriously affect the MD resonant response of the structure. Considering the extensive possible combinations of these parameters, we take the step-by-step optimization method to orderly optimize the surface. In the first optimization process, the element spaces \( d_1 \) and \( d_2 \) are optimized first. We consider both the equal spacing condition, \( d_1 = d_2 \), and the unequal spacing condition, namely \( d_1 \neq d_2 \). After numerical calculation, the best result of linear phase profile occurs when \( d_1 = d_2 = 0.86 \) \( \mu m \). The derived parameter configuration is represented as configuration 2. The sketches of configurations 1 and 2 are plotted in Figs. 6(a) and 6(b), respectively, for intuitive understanding. The phase map of configuration 2 is plotted in Fig. 6(d). Compared with the phase profile of configuration 1 in Fig. 5(c), Fig. 6(d) shows a better phase gradient changing profile. A roughly linear phase-changing profile can be observed. In Fig. 5(f), we plot the transmission of configuration 2. At 1.495 \( \mu m \), 82\% transmission is achieved for configuration 2.

Based on configuration 2, the parameter \( \lambda_x \) of the three elements is further optimized. A sketch in Fig. 6(c) summarizes the

Table 1. First Configuration of the Phase Gradient Metasurface

<table>
<thead>
<tr>
<th>Element</th>
<th>( \lambda_x ) (( \mu m ))</th>
<th>Transmission</th>
<th>Phase (rad)</th>
<th>Ideal Phase (rad)</th>
<th>Real Phase conf. 1</th>
<th>Real Phase conf. 2</th>
<th>Real Phase conf. 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>0.24</td>
<td>0.87</td>
<td>2.10</td>
<td>2.09 (2( \pi/3 ))</td>
<td>3.42 (rad)</td>
<td>2.17 (rad)</td>
<td>2.12 (rad)</td>
</tr>
<tr>
<td>E2</td>
<td>0.39</td>
<td>0.97</td>
<td>4.21</td>
<td>4.19 (4( \pi/3 ))</td>
<td>0.575 (rad)</td>
<td>5.10 (rad)</td>
<td>4.31 (rad)</td>
</tr>
<tr>
<td>E3</td>
<td>0.43</td>
<td>0.97</td>
<td>6.23</td>
<td>6.28 (2( \pi ))</td>
<td>0.81 (rad)</td>
<td>6.20 (rad)</td>
<td>6.15 (rad)</td>
</tr>
</tbody>
</table>

*Transmission and phase refer to the transmissive and phase response of the corresponding periodically arranged element arrays with a periodicity of 1 \( \mu m \). The real phase is the phase measured inside the metasurface.*

![Figure 5](image)
The difference between configurations 2 and 3. The optimized phase profile is plotted in Fig. 6(e). Compared with the two former results, this latest result has a better linear phase-changing effect. In Fig. 5(f), the transmission of configuration 3 is 83.5% at 1.521 μm, where the best linear phase profile is achieved. These results indicate that the optimization processes are very effective and necessary in our designing scheme. According to our simulation, the two-step optimization processing can be taken inversely. The same result as Fig. 6(e) can also be realized by inverting the two optimization processes. The real phase response of the elements under configurations 2 and 3 are listed in Table 1. Clearly, the real phase response approaches the ideal phase response when the optimization processes are imposed on the metasurface from configuration 1 to configurations 2 and 3.

In our two-step optimization process, the element–parameter optimization corrects the originally distorted phase response of the three elements inside the metasurface unit cell. The optimized three elements present near-ideal phase responses. The element-spacing optimization can improve the linearity of the metasurface phase profile. To better show the effect of the element-spacing optimization, we plot the transmissive response and phase profile of the surface varying with the element spacing \( d \) in Figs. 7(a) and 7(b). For simplicity, we only show the result at the condition \( d = d_1 = d_2 \). According to Fig. 7(b), the structure can realize a 2\( \pi \) phase shift at any element spacing \( d \), but the corresponding linearity of the 2\( \pi \) phase shift is varied. This indicates that the element-spacing variation has limited change on the phase profile of the designed metasurface. However, in Fig. 7(a), we can see that the transmission of the metasurface changes from 40% to near 90% with the variation of the element spacing \( d \). This result implies that the efficiency of the metasurface can be efficiently optimized by adjusting the element spacing. The adjusting of the element spacing leads to constructive or destructive interference to the elements. According to Figs. 7(a) and 7(b), the interference seriously affects the transmissive response for the metasurface.

According to numerical calculation, the anomalous deflecting angle of the proposed metasurface is about 30°, and the transmission is 83.5%. In Fig. 8, far-field transmitting intensities are

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**Fig. 6.** (a) Top view of the metasurface configuration 1. (b) Top view of the metasurface configuration 2. The parameters of each element stay unchanged, while only the spaces \( d_1 \) and \( d_2 \) between three elements change from 1 to 0.86 μm. (c) Top view of the metasurface configuration 3. Element parameters \( l_x \) are optimized. The optimized parameters \( l_x \) of the three elements are 0.3, 0.4, and 0.56 μm, respectively. (d) Calculated phase distribution of the second configuration of the metasurface; the operating wavelength is 1.495 μm. (e) Calculated phase distribution of the third configuration of the metasurface; the incident wavelength is 1.521 μm.

**Fig. 7.** (a) Transmissive response of the metasurface varying with the element spacing \( d \). Black dotted line marks the applied working wavelength. (b) Phase response along the \( x \) direction inside a metasurface unit cell varying with the element spacing \( d \) at working wavelength. The parameters \( l_x \) of the three elements are 0.3, 0.4, and 0.56 μm, respectively.
plotted under the three configurations. Apparently, the light deflected into the ±2, 0, and −1 diffraction orders is well suppressed with the optimizing processing from configuration 1 to 3. The final result, namely the third configuration in Fig. 8(c), contains a major +1 ordered diffracted light and minor −1 and 0 ordered diffracted light. In Figs. 8(a)–8(c), the transmitting intensity is normalized to the total transmitting intensity. Our calculation shows that the sum of the transmitting intensity at around +1 ordered diffraction possesses 93% of the total transmitting intensity, while the total transmitting intensity of −1 and 0 ordered diffraction together possesses only 7% of the total transmitting intensity.

According to the generalized Snell’s law [1], the angle of anomalous refraction can be theoretically predicted by
\[
\theta_t = \arcsin \left( \frac{1}{n_t} \left( \frac{\lambda_0}{P} + (\sin \theta_i)n_i \right) \right),
\]
(6)
where \( n_i \) and \( n_t \) are the refractive index of the transmitting and incident environments, respectively; \( P \) is the periodicity of the array in the phase-gradient direction; and \( \lambda_0 \) is the operating wavelength. In our situation, the incident and transmitting environments are set as air, namely \( n_i = n_t = 1 \). The influence of the substrate and coating layer on the diffraction angle can be neglected, as their thicknesses are less than half of the incident wavelength. At the normal incidence condition, the incident angle \( \theta_i \) is 0°, the operating wavelength is 1.521 μm, and the periodicity is 3 μm in the \( x \) direction. The theoretically calculated anomalous deflection angle is 30.5°, which is in good agreement with the simulation result (30°). Compared with many other Huygens’ phase-gradient metasurfaces, the proposed metasurface has a larger deflecting angle while the phase-change linearity is well. The excellent performance is likely owing to two main reasons. The first is that the number of the surface unit cell elements is small; only three elements are applied to build the phase gradient surface. Another reason is that the element-spacing optimization process is used to better control the phase profile of the metasurface.

According to Eq. (6), the anomalous refraction angle can be tuned by changing the periodicity of the metasurface. However, the periodicity of Huygens’ metasurface influences the interaction between Huygens’ elements. A large adjustment of periodicity can easily bring about distortion for the phase distribution of the metasurfaces. Although minor adjustment of periodicity maintains the functionality of the metasurface, the anomalous refraction angle can only be tuned in a very limited region (less than 1°). An efficient way to realize different anomalous refraction angles is to add elements to expand the periodicity of the metasurface. Therefore, we further design metasurfaces with four, five, and six Huygens’ elements. The optimized results are plotted in Fig. 9, showing that near-linear

![Fig. 8. Far-field transmitting intensity (T.I.) for the three configurations at different diffraction angles. The transmitting intensity is normalized to the total transmission intensity.](image)

![Fig. 9. Phase distribution of the Huygens’ metasurfaces consisting of different element numbers. (a) Four-element Huygens’ metasurface. (b) Five-element Huygens’ metasurface. (c) Six-element Huygens’ metasurface. Each figure consists of two unit cells to better show the anomalous deflection effect. The spacing between neighboring elements in these three configurations is set as 0.86 μm for simplicity. Other parameters are listed in Table 2.](image)
phase profiles can be realized in four, five, and six-element Huygens' metasurfaces. The corresponding parameters and detailed numerical results are listed in Table 2. The refraction angles range from 30° to 15.8°. In our optimizing process, the element geometric parameter $l_x$, periodicity $P_n$, and space $d$ between neighboring elements are the main optimizing parameters. A shortcoming of the proposed optimization method is that the optimizing parameters increase with the number of elements. Therefore, the optimizing process is too complex when all the optimizing parameters are considered. These results prove that our design strategy is effective for more than one specific case.

### 3. CONCLUSION

In this paper, we propose an all-dielectric three-element Huygens' metasurface realizing anomalous refraction based on cross-shaped structures. Utilizing the multipole decomposition method, we derive the ED and MD resonant wavelengths and intensities inside cross-shaped structures. We find that the ED resonance shows a more active response to the change of the $x$-directed arm of the cross particle on resonant intensity and wavelengths than the MD resonance. Utilizing this performance, we realize the spectral overlapping between the ED and MD resonances. We prove that the traditionally fixed element spacing inside a metasurface can be an efficient phase-profile-adjusting parameter. Through a two-step optimization process, the traditionally designed Huygens' metasurface can realize desired anomalous refraction. The proposed three-element metasurface can refract a normal TM incident beam into a 30° oblique transmissive beam with a transmissive efficiency of 83.5%.

We also design metasurfaces with different element numbers according to our design strategy. By increasing the element number in the metasurface, the anomalous refractive angle can be tuned in the range from 15.8° to 30°. Our results prove that functional Huygens' metasurfaces with small element numbers can be efficiently realized through element–element spacing control. The methods and solutions used in this paper can help to design other high-efficiency metasurfaces.

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### REFERENCES


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Table 2. Crucial Parameters of the Huygens’ Metasurfaces Consisting of Different Numbers of Huygens’ Elements

<table>
<thead>
<tr>
<th>E. N.</th>
<th>$l_x$</th>
<th>$l_y$</th>
<th>$l_z$</th>
<th>$l_x$</th>
<th>$l_y$</th>
<th>$l_z$</th>
<th>$P$</th>
<th>$\lambda_0$</th>
<th>$\theta_t$</th>
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<tr>
<td>3</td>
<td>0.3</td>
<td>0.4</td>
<td>0.56</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>3</td>
<td>1.521</td>
<td>30°</td>
</tr>
<tr>
<td>4</td>
<td>0.27</td>
<td>0.38</td>
<td>0.42</td>
<td>0.56</td>
<td>–</td>
<td>–</td>
<td>3.8</td>
<td>1.506</td>
<td>23.4°</td>
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<tr>
<td>5</td>
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<td>0.35</td>
<td>0.38</td>
<td>0.4</td>
<td>0.5</td>
<td>–</td>
<td>5</td>
<td>1.495</td>
<td>17.4°</td>
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<td>0.38</td>
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<td>0.56</td>
<td>5.6</td>
<td>1.524</td>
<td>15.8°</td>
</tr>
</tbody>
</table>

*E. N. represents element number.

These authors contributed equally to this work.


