Discrete spatial solitons formed in periodically poled lithium niobate by electro-optical effect

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We report the numerical observation of discrete spatial solitons in a periodically poled lithium niobate waveguide array by applying an electrical field through electro-optical effect. We show that discrete spatial soliton can be controlled by applied voltage in the periodically poled lithium niobate.

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Since the predication of spatial optical soliton\(^{[1,2]}\), it has attracted more and more attention because of its physical interests and potential applications in all-optical processes. The spatial optical soliton is formed when the nonlinear optical effect balances exactly the diffraction in Kerr media under the assumption that the propagation loss can be neglected, and the light beam propagates along the nonlinear optical material accordingly with its transverse profile unchanged. Until now, many studies are focused on the contributions of different nonlinear effects to the formation of spatial soliton, for example, saturable nonlinear effects\(^{[3,4]}\), photo-refractive effects\(^{[5]}\) and that in quasi-phase-matching (QPM) materials\(^{[6]}\) respectively. On the other hand, one can assume that the spatial soliton should exist in nonlinear optical media when the self-focusing induced by nonlinear optical effect can be balanced by other spatial broadening effects besides diffraction effect. In the waveguide array, the weak light incident into a waveguide can be coupled to its neighboring waveguides as it propagates. Christodoulides\(^{[7]}\) first theoretically predicted the spatial soliton in the waveguide array in which Kerr-induced spatial focusing effect is balanced by the widening distribution by waveguides coupling. They called it “discrete spatial soliton”\(^{[8]}\).

Recently, experimental demonstrations of this kind of spatial optical soliton have been done in 41-waveguide array in an AlGaNAs substrate\(^{[9]}\).

Periodically poled lithium niobate (PPLN) is used for frequency conversion by QPM\(^{[10]}\). When a fundamental wave is loosely focused into the PPLN crystal, along the $x$-axis as shown in Fig. 1, the second-harmonic generating (SHG) wave continuously increases when the phase mismatch between the pump and SHG waves is compensated by the spatial reciprocal vector of the domain-inverted gratings. However, besides the $\chi^{(2)}$ nonlinear coefficient, other second-rank tensors, such as electro-optical coefficient, are also modulated periodically in the PPLN crystal due to periodic domain inversion. Some applications, such as electro-optical lens, deflector and wavelength filter were suggested based on index difference by electro-optical effect in opposite domain regions.

When an electrical field is applied along the $z$-axis in Fig. 1, index grating by electro-optical effect is generated. If the light is incident perpendicularly to the $x$-$z$ plane, it sees waveguide array!

In this paper, we first study the waveguide array generated by electro-optical effect, the coupling between the waveguides as functions of electrical field and domain structure is analyzed in detail. Based on the studies of waveguides coupling in PPLN, a discrete spatial soliton is predicted. Compared with that in AlGaNAs materials, discrete spatial soliton in PPLN waveguide array shows its advantages in easy controllability and flexibility.

When an external electrical field is applied along the $\epsilon$-axis of PPLN crystal, the extraordinary refractive index will change due to the linear electro-optical or Pockels effect. The positive and the negative domain have opposite response to the strength of the applied electrical field because the electro-optical coefficient in the negative domain region changes its sign after domain inversion. As shown in Fig. 1, when a voltage is applied along $z$-axis, the indexes of the positive and negative domain are give as follows, respectively

\[
\begin{align*}
\begin{cases}
 n_p = n_e - \frac{1}{2} n_e^2 \cdot r_{33} \cdot \frac{U}{t}, & \text{for positive domain} \\
 n_n = n_e + \frac{1}{2} n_e^2 \cdot r_{33} \cdot \frac{U}{t}, & \text{for negative domain}
\end{cases}
\end{align*}
\]

where $n_e$ is refractive index of LiNbO$_3$ for TE input light, $r_{33}$ is its electro-optical coefficient, $U$ is applied voltage in z-axis, and $t$ is thickness of the crystal.

If a light propagates along the $z$-axis, the modulation of the index can be regarded as an index grating, in QPM nonlinear optical process such as optical parametric oscillation (OPO), the wavelength of output light can be tunable by applying external electrical field\(^{[11]}\). On the other hand, if a light propagates along the $y$-axis, the index modulation will behave like an infinite array of identical, weakly coupled waveguides when the external voltage is applied.

When a light is focused into one waveguide, it will couple to more and more waveguides as it propagates like

![Fig. 1. Schematic diagram of periodic domain inverted LiNbO$_3$ with an applied voltage along z-axis.](http://www.col.org.cn)
directional coupler, broadening its spatial distribution. This kind of distribution is analogous to diffraction in continuous media. All the guides support the same optical mode and energy can be exchanged between neighboring guides through the overlap of their modes. According to the coupling mode equations\(^{11}\), the coupling coefficient of the waveguide array is

\[
C = \int \phi(x)(n_y^2 - n_y^2)\phi^*(x + \Lambda)dx
\]

\[
= \frac{2k^2e^{-\pi d}}{(k^2 + p^2)\left(t + \frac{2}{\beta}\right)}.
\]

(2)

where \(\phi(x)\) is the normalized mode field of the waveguide, \(\beta\) is propagation constant, \(w\) and \(d\) are width of neighboring domains, and \(p\) and \(k\) are propagation constants in transverse directions in neighboring domain, respectively. The coupling coefficient is an overlap integral between waveguide mode field and index difference, both of the two contributions are dependent on the applied electrical field. The relation of the coupling coefficient \(C\) and the electrical field \(E\) with various values of the ratio \(w:d\) are given in Fig. 2. From Fig. 2, it is clear that the coupling coefficient has a maximum value within the range of electrical field we calculated. When the applied voltage increases, the index difference \(n_y^2 - n_y^2\) increases, making a positive contribution to the overlap integral. In the mean time, the guide mode confinement of the waveguide becomes stronger, so that the overlap integral of mode fields between the two neighboring waveguides decreases as the increase of the applied voltage. When the external electrical voltage is small, the contribution of the index difference dominates, until the coupling coefficient reaches its maximum. After that, the contribution from the mode overlap integral exceeds that of the index difference, so that the coupling coefficient of waveguide array decreases as the applied electrical field increases. Because the slope of climbing region of the curve is much larger than that of the decline region, the coupling coefficient is more sensitive to the applied voltage in the climbing region. A 1-kV/mm change of the applied field is enough to double the coupling coefficient from the curves shown in Fig. 2. In addition, we found that the electrical field corresponding to the maximum of coupling coefficient is almost same when the period of waveguide array in PPLN is fixed.

We also study the coupling coefficient as a function of the period \(\Lambda\) of waveguide as shown in Fig. 3. The coupling coefficient decreases as the period increases for the same ratio of \(w:d\) because the overlap integral of the neighboring mode fields becomes small.

When the intensity of input light is increased, the incident beam tends to be brought to a focus by means of the cubic nonlinearity of the material. As a result of this nonlinear response, the refractive index of the material is larger at the center of the laser beam than at its periphery, and the medium is in effect turned into a positive lens. When the power of input light surpasses the critical power and the tendency of the beam to spread due to coupling in waveguides is precisely compensated by the tendency of the beam to contract due to self-focusing in nonlinear material, a phenomenon known as self-trapping may occur and the fixed spatial profile among a number of waveguides in PPLN will be kept over a long distance, like the spatial solitons, called discrete spatial solitons.

Considering only nearest neighbor interaction, we can find discrete spatial solitons which are solutions of the discrete nonlinear Schrödinger equation, a kind of partial differential equation with solitary solutions. The electrical field propagating in the \(n\)th waveguide obeys the following difference differential equation\(^{12,13}\)

\[
i^2
\frac{dE_n}{dz} + \beta E_n + C(E_{n+1} + E_{n-1}) + \lambda |E_n|^2 E_n
\]

\[+ \mu (|E_{n+1}|^2 + |E_{n-1}|^2)E_n = 0,
\]

(3)

where \(\lambda = \frac{\omega_0^2}{\Lambda_\text{eff}}\), where \(\omega_0\) is the optical angular frequency of input light, \(\Lambda_\text{eff}\) is the common effective area of the waveguide modes and the last term of Eq. (3) arises from the nonlinear overlap of the adjacent modes. As pointed out in previous studies, the term \(\lambda |E_n|^2E_n\) describing self-focusing effect dominates the nonlinear process, that is \(\lambda \gg \mu\), and hence we set \(\mu = 0\) in the later discussions.

If the intensity varies slowly over adjacent waveguides (in the \(z\)-axis), it is possible to study the Eq. (3) in the long-wavelength approximation or in the so-called continuum approximation. It has been shown numerically that when the input power exceeds the critical value and only one guided-mode can propagate in waveguide, the fundamental soliton or self-focusing solution is given by

![Fig. 2. Diagram of coupling coefficient as a function of applied electrical field. The period of waveguide array is 6 \(\mu\)m.](image1)

![Fig. 3. Diagram of coupling coefficient versus period of waveguide. The applied electrical field is 2.4 kV/mm.](image2)
\[ E_n(z) = A_0 \exp[i(2C + \beta)z] \sec h \left( \frac{x_n}{x_0} \right) , \] (4)

where \( A_0 \) is equal to \( \sqrt{\frac{P_0}{\lambda}} \) according to the previous study, and \( p \) is a dimensionless parameter. The above equation can be transformed to

\[ E_n(z) = \sqrt{\frac{P_0}{\lambda}} \exp[i(2C + \beta)z] \sec h \left( \frac{x_n}{x_0} \right) . \] (5)

The spatial profile of discrete spatial soliton is shown in Fig. 4. From Eq. (5) and the expression \( \lambda = \frac{\omega_0 z}{C_{\text{ave}}} \), peak power required for a discrete spatial soliton is proportional to the square of the coupling coefficient between neighboring waveguides.

Figure 5 shows the relative peak power for discrete spatial soliton as the function of electrical field for different waveguide spacings. The narrower the waveguide spacing and the shorter the coupling length, the higher power required for discrete spatial soliton is. To achieve a trade-off between the peak power and the coupling length required for a soliton in PPLN, it is in view of practical uses, low soliton power and short coupling length are appreciated.

It is interesting to compare our theoretical studies in waveguide array by electro-optical effect in PPLN with that in AlGaAs waveguide array. In Ref. [8], the value \( \lambda = \frac{\omega_0 z}{C_{\text{ave}}} \), describing the nonlinear coefficient, is 3.6 m\(^{-1}\)W\(^{-1}\), the coupling coefficient is 0.8 mm\(^{-1}\), and the corresponding peak power for discrete spatial soliton is almost 500 W. In our model, when 2-kV/mm electrical field is applied on the PPLN with the waveguide spacing of 1 \( \mu \)m and the period of 6 \( \mu \)m, the value \( \lambda = \frac{\omega_0 z}{C_{\text{ave}}} \) is calculated to be about 0.12 m\(^{-1}\)W\(^{-1}\), the coupling coefficient is 0.03 mm\(^{-1}\), and the peak power for discrete spatial soliton is about 500 W which is nearly the same as that in AlGaAs waveguide array\(^{[8]}\).

As we know, the cubic nonlinear coefficient of lithium niobate is much smaller than that of AlGaAs material, so that longer crystal and wider spacing between neighboring waveguides are necessary in order to form a discrete spatial soliton with almost same peak power. Because waveguide array in PPLN crystal is formed by applying external field, the coupling coefficient and the peak power for the soliton are electrical field dependent. Spatial light distribution from the PPLN waveguide array can be modulated by adjusting the applied voltage.

![Fig. 4. Numerical result of spatial profile of discrete spatial soliton in PPLN waveguide array.](image)

Experimental demonstrations of discrete spatial solitons in PPLN controlled by electrical field are in progress.

The waveguide array can be established by applying an electrical field along the c-axis of PPLN. Coupling coefficients as the functions of the applied voltage and the domain inversion parameters of PPLN are calculated to provide a theoretical basis for the discrete spatial soliton in PPLN waveguide array. It is shown that discrete spatial soliton depends on the applied field and also the structure of the domain inversion. The comparison between the discrete spatial solitons in AlGaAs and PPLN waveguide array is proposed in order to certify the feasibility of the soliton in PPLN by electro-optical effect.

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