The reconstruction of diffractive object digital hologram at a short distance

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By using a spherical wave as the reference wave, we recorded the in-line phase-shifting digital hologram of the 25th element of Chinese standard No. 3 resolution test pattern, and gave the corresponding numerical reconstructed results. Some problems concerning with the digital hologram recording and reconstruction of the diffractive object at a short distance are discussed. The experimental result shows that the resolution of the reconstructed image is better than 10 μm, which is the limit by using this experimental arrangement. OCIS codes: 090.1760, 190.0180, 100.3010.

The character of holography is to record and reconstruct the phase of the object\(^1\). In comparison with classical holography, one of the advantages of digital holography is that the complex amplitude of the object can be directly reconstructed, which will supply an extensive application in many fields. However, because of the low sampling frequency and the small imaging area of charge coupled device (CCD), a major problem in digital holography is to improve the resolution of the reconstructed image. There are two methods to improve the resolution of digital holography: one is to shorten the distance between the object and CCD\(^2\), and the other is to adopt the synthesize aperture to enlarge the imaging area of CCD\(^3\). To fulfill Nyquist sampling condition and the separating problem of the reconstructed images, it is very important to use a proper digital holographic recording optical arrangement. By use of in-line Fourier digital holography\(^4\), Nyquist sampling condition can be fulfilled easily. Further more, by using the phase-shifting technique, the separating problem of the reconstructed images in in-line Fourier digital holography can be solved ideally\(^5,6\).

The diffraction angle of the microscopic object is large, and the distribution of the amplitude and phase of the diffraction field is very complex. In order to achieve the high quality reconstructed image and improve the resolution, we should adjust the recording distance to ensure the zero- and first-order diffraction information to be recorded leastwise. If the recording distance is too long, the microstructure of the object cannot be recorded. However, if the recording distance is too short, the Nyquist sampling condition is not fulfilled in some high frequency sections, and the resolution of the reconstructed image also cannot be improved.

In microscopic in-line digital holography, if a spherical wave is used as the reference wave, the point-source center of the reference wave is located on the optical axis of the system. By adjusting the distance between the reference wave and the object, Nyquist sampling condition can be easily satisfied. Moreover, by adopting phase-shifting technology, the signal-to-noise ratio (SNR) of digital hologram will be improved and the reconstructed conjugate images can be eliminated easily.

Figure 1 shows the coordinate system of our experimental configuration for recording in-line phase-shifting digital hologram. The object plane and the hologram plane are denoted by \((x_0, y_0)\) and \((x_\text{II}, y_\text{II})\), respectively, the distance between the object plane and the hologram plane is defined as \(z_\text{OH}\). If Fresnel diffraction condition is fulfilled, the complex amplitude distribution of the object wave in the CCD plane can be represented as

\[
O(x_\text{II}, y_\text{II}) = C_{\text{OH}} \int O(x_0, y_0) \times \exp[ik(x_0^2 + y_0^2)/(2z_{\text{OH}})] \times \exp[-j2\pi(x_0 x_\text{II}/\lambda z_{\text{OH}} + y_0 y_\text{II}/\lambda z_{\text{OH}})] dx_0 dy_0, \tag{1}
\]

where

\[
C_{\text{OH}} = \frac{\exp(i k z_{\text{OH}})}{j \lambda z_{\text{OH}}} \times \frac{\exp[i k(x_\text{II}^2 + y_\text{II}^2)/(2z_{\text{OH}})]}{j \lambda z_{\text{OH}}} \tag{2}
\]

Assume the point-source center of the reference wave is located in \((0, 0, z_{\text{RH}})\), as shown in Fig. 1, then the complex amplitude distribution of reference wave in the CCD plane is described as

\[
R(x_\text{II}, y_\text{II}) = \frac{\exp[i k(z_{\text{RH}} + (x_\text{II}^2 + y_\text{II}^2)/(2z_{\text{RH}}))]}{z_{\text{RH}}} \tag{3}
\]

We use four-step phase-shifting technique to eliminate the reconstructed conjugate images. If the phase of the reference wave was shifted by 0, \(\pi/2\), \(\pi\), 3\(\pi/2\), respectively, we recorded four corresponding in-line holograms \(H_n(n = 1, 2, 3, 4)\). Multiplying the four holograms by \(\exp[i(n - 1)\pi/2]\) respectively, and then adding them together, we can obtain a four-step phase-shifting technique for recording in-line phase-shifting digital hologram.

![Figure 1. The coordinate system for recording digital hologram.](http://www.col.org.cn)
hologram[6]

\[ I(x_H, y_H) = \sum_{n=1}^{4} I_n \exp[j(n-1)\pi/2] \]
\[ = 4O(x_H, y_H)R^*(x_H, y_H). \] (4)

Illuminating the four-step phase-shifting digital hologram by a reconstructed wave that is same as the experimental reference wave, and adjusting the image plane to be overlapping with the object plane \((z_{OH} = z_{OH})\), then the complex amplitude distribution of the reconstructed wave can be written as

\[ O(x, y) = C_{OH} \iint I(x_H, y_H) \times \exp \left[ jk \frac{(x_H^2 + y_H^2)}{2} \times \left( \frac{1}{z_{RH}} - \frac{1}{z_{OH}} \right) \right] \times \exp \left[ j2\pi \left( \frac{x}{\lambda z_{OH}} + \frac{y}{\lambda z_{OH}} \right) \right] dx_H dy_H, \] (5)

where

\[ C_{OH} = j \exp \left\{ jk \left[ (z_{RH} - z_{OH}) - \frac{(x^2 + y^2)}{(2z_{OH})} \right] \right\} \frac{4\lambda z_{OH} z_{RH}}{4\lambda z_{OH} z_{RH}}. \] (6)

It can be seen that Eq. (5) is the inverse algorithm of the object diffraction. According to holographic theory and the experimental results of holographic interferometry[7], we can derive that the reconstructed real image is the same as the recorded object.

Figure 2 shows the experimental configuration for recording in-line phase-shifting digital microscopic hologram. A Chinese standard No. 3 resolution test pattern is used as the recorded object. A collimated beam of He-Ne laser \((\lambda = 632.8\, \text{nm})\) is split into two beams by a splitter BS1. One of the two split beams illuminates the object after being reflected by the reflector M2. The other beam is split into two beams by another splitter BS2, and the reflecting wave, regarded as the reference wave, is reflected and phase-shifted by a piezoelectric transducer (PZT) which is fixed behind the reflector M1, and then transmits through the splitter BS3 and BS3. The object wave and the reference wave are combined in the CCD plane, and the in-line hologram is recorded by the CCD. The CCD has a sensor array of \(793(\text{H}) \times 596(\text{V})\) pixels, and the size of each pixel is \(0.0100 \times 0.0108\, \text{mm}^2\). Figure 3(a) shows a recorded hologram of the 25 element of the resolution test pattern, whose size is \(7.68 \times 6.22\, \text{mm}^2\). Figure 3(b) is the amplitude distribution of a four-step phase-shifting hologram which is calculated in terms of Eq. (4). According to Eq. (5), Fig. 4(a) gives the reconstructed image of the resolution test pattern and Fig. 4(b) gives the intensity distribution of a section of the vertical pattern in Fig. 4(a).

In Fig. 3(b), we can see not only the first order diffraction patterns of each pair of lines, but also that of the corresponding rectangular aperture, and the SNR of the hologram is improved clearly. The distance between the zero- and first-order diffraction is \(6.263\, \text{mm}\), which is calculated from the four-step phase-shifting digital hologram, and the recording distance is \(98.97\, \text{mm}\), which is derived from the formula of the diffraction grating.

When the reconstructed distance \(z_{OH} = -99.0\, \text{mm}\) and the distance between the reconstructed point source

![Fig. 2. Experimental configuration for recording an in-line phase-shifting digital microscopic hologram.](image-url)
center and the hologram plane $z_{RH} = 81.0$ mm, the section which contributes to the reconstructed real image is about $1.2 \times 0.82$ mm$^2$. As shown in Figs. 4(a) and (b), the zero-order image and the conjugate image are eliminated completely, and the reconstructed image with 10 $\mu$m-wide line can be distinguished evidently, which is the diffraction limit resolution of this experimental arrangement. Meanwhile, as the recording distance is very short, if the object position or the point source center position of the reference wave changes for 1%, the quality of the reconstructed image will be influenced greatly. In addition, in order to improve the quality of the reconstructed image, we also corrected the slight astigmatism of the reference wave which comes from the cuneal beam splitter in reconstruction.

This paper gives the results for recording and reconstruction of the phase-shifting in-line digital hologram of a Chinese standard No. 3 resolution target in the Fresnel diffraction condition, and the resolution of the reconstructed image achieves 10 $\mu$m which is the diffraction limit of this experimental system. In order to achieve high quality reconstructed image and apply the digital holographic technique into quantitative calculation, if the recording distance is very short, two factors must be thought: Firstly, the distance between the object and CCD, and the distance between the point source center of the reference wave and CCD, should be measured precisely. Secondly, the reference wave must be an ideal spherical wave. Furthermore, if the recording distance is much more shortened, so that Fresnel diffraction condition is not fulfilled, the method for digital hologram recording and reconstruction should be discussed in other way.

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