The novel characteristics of guided modes in the chiral negative refractive index fiber are investigated theoretically in this letter. We derive the characteristic equation of guided modes. Based on two types of chiral metamaterial parameters, we present the dispersion curves, energy flux, and power of guided modes. Some abnormal features, such as the existence of surface mode and dispersion curves with different shapes, intersection of dispersion curves of different guided modes, negative energy flux in the achiral cladding, and zero power at some normalized frequencies, are found in the chiral negative refractive index fiber.

From the constitutive relations and Maxwell’s equations, the longitudinal components of the electromagnetic fields in the core and cladding can be solved as follows\cite{30}.

In the core \((0 \leq \rho \leq a)\):
\[
\begin{align*}
E_{1z} &= [A_m J_m(h_1+\rho)+B_m J_m(h_1-\rho)] \exp(jm\varphi) \\
H_{1z} &= \frac{1}{n_1}[A_m J_m(h_1+\rho)-B_m J_m(h_1-\rho)] \exp(jm\varphi)
\end{align*}
\]

in the cladding \((\rho \geq a)\):
\[
\begin{align*}
E_{2z} &= C_m K_m(h_2\rho) \exp(jm\varphi) \\
H_{2z} &= \frac{1}{\eta_2} D_m K_m(h_2\rho) \exp(jm\varphi)
\end{align*}
\]

The constitutive relations in an isotropic chiral medium for a time-harmonic field with \(\exp(j\omega t)\) can be written as\cite{4,29}
\[
D = \varepsilon E - j\kappa \sqrt{\mu_0} \varepsilon_0 H, \quad B = \mu H + j\kappa \sqrt{\mu_0} \varepsilon_0 E.
\]

\[
\begin{align*}
E_{1z} &= [A_m J_m(h_1+\rho)+B_m J_m(h_1-\rho)] \exp(jm\varphi) \\
H_{1z} &= \frac{1}{n_1}[A_m J_m(h_1+\rho)-B_m J_m(h_1-\rho)] \exp(jm\varphi)
\end{align*}
\]
The transverse components of the electromagnetic fields $E_{1\varphi}, H_{1\varphi}, E_{2\varphi}, H_{2\varphi}$ can be derived from the relations between longitudinal and transverse components of the electromagnetic fields.\(^{[20]}\) Then, imposing the conditions of continuity of $E_z, E_{\varphi z}, H_z, H_{\varphi z}$ at $\rho = a$, the characteristic equation of guided modes can be obtained as

$$a_{11}a_{22} - a_{12}a_{21} = 0,$$

where

$$a_{11} = \left( \frac{1}{u_{a}^2} + \frac{1}{v_{a}^2} \right) \beta_a J_m(u_{a}) - \frac{k_{a}a}{u_{a}^2} J_m(u_{a}) - \frac{k_{a}a}{v_{a}^2} J_m(u_{a}) \hat{K}_m,$$

$$a_{12} = \left( \frac{1}{u_{a}^2} + \frac{1}{v_{a}^2} \right) \beta_a J_m(u_{a}) + \frac{k_{a}a}{u_{a}^2} J_m(u_{a}) + \frac{k_{a}a}{v_{a}^2} J_m(u_{a}) \hat{K}_m,$$

$$a_{21} = \left( \frac{1}{u_{a}^2} + \frac{1}{v_{a}^2} \right) \beta_a J_m(u_{a}) - \frac{k_{a}a}{u_{a}^2} J_m(u_{a}) - \frac{k_{a}a}{v_{a}^2} J_m(u_{a}) \hat{K}_m,$$

and $u_{\pm} = h_{1,\pm}a, v = h_2a, \eta = \frac{u_{\pm}}{v_{a}}$. $\hat{K}_m = K_m(v)$. In the core, this phenomenon has also been found in energy flux along the $z$-axis in the waveguide is defined by

$$S_z = \frac{1}{2} \text{Re}(E \times H^*) \cdot \hat{z} = \frac{1}{2} \text{Re}(E_{\varphi}H_{\varphi}^* - E_{\varphi}H_{\varphi}^*).$$

Powers in the core ($P_1$) and cladding ($P_2$) are the integration of the energy flux $S_{z1}$ and $S_{z2}$, respectively:

$$P_1 = \int_{0}^{2\pi} \int_{0}^{a} \rho S_{z1} \rho \varphi \, d\rho \, d\varphi = 2\pi \int_{0}^{a} \rho S_{z1} \rho \, d\rho,$$

$$P_2 = \int_{0}^{2\pi} \int_{a}^{\infty} \rho S_{z2} \rho \varphi \, d\rho \, d\varphi = 2\pi \int_{a}^{\infty} \rho S_{z2} \rho \, d\rho.$$ 

The normalized power is defined as\(^{[25]}\)

$$P = \frac{P_1 + P_2}{|P_1| + |P_2|}.$$ 

Now we present the numerical results for two types of chiral metamaterial parameters that both satisfy $n_2 < |n_{1-}| < n_{1+}$: $n_1 > n_2$ and $n_1 < n_2$. Here we use normalized frequency $V = k_0a\sqrt{|n_1^2 - n_2^2|}$ because the chiral metamaterial occurs only at a certain frequency.

Consider the first case: $n_1 > n_2$. We choose $\varepsilon_1 = 2.25\varepsilon_0, \mu_1 = \mu_0, \varepsilon_2 = \varepsilon_0, \mu_2 = \mu_0, \kappa = 2.6,$ and refractive index parameters $n_{1+} = 4.1, n_{1-} = -1.1, n_1 = 1.5,$ and $n_2 = 1.0$, which satisfy $n_{1-} < 0$ (chiral negative refractive index), $n_1 > n_2$, and $n_2 < |n_{1-}| < n_{1+}$.

Figure 2 shows the normalized propagation constant $\beta/k_0$ versus normalized frequency $V$ for modes (a) $m = 0$, (b) $m = -1, 1$, and (c) $m = -2, 2$, in the chiral negative refractive index fiber for $n_1 > n_2$. Solid curves correspond to $m = 0, -1, -2$ and dashed curves correspond to $m = 1, 2$, where $H_{mn}$ and $H_{mn}$ stand for guided modes and surface modes, respectively.

Figure 2 shows some novel characteristics of dispersion curves as follows.

1) Mode bifurcation (i.e., propagation constants are different for different signs of $m$). The cutoff frequencies of $m = -1, 1, -2, 2$ guided modes are the same, which satisfy the equation $J_1(\sqrt{k_0^2 + k_2^2}) = 0$. However, the cutoff frequencies of $m = 2, 2$ guided modes are different. The mode bifurcation phenomenon occurs in the conventional chiral fibers (with small chirality parameters)\(^{[23,24]}\); however, it cannot occur in negative refractive index fibers\(^{[25,26]}\).

2) Dispersion curves of $m = -1, -2$ guided modes are bent, with their shapes similar to the letter “S” for the second- and other higher-order guided modes. In some range of $V$, there are two or three values of propagation constants for a fixed $V$, and the slope of dispersion curves can be negative. Similar dispersion curves can be found in negative refractive index fibers\(^{[25]}\). In the conventional chiral fibers\(^{[23,24]}\), the slopes of dispersion curves are always positive.

3) There are several mode intersections, such as those in guided modes of $H_{-12}$ and $H_{12}, H_{-13}$ and $H_{13}, H_{-22}$ and $H_{22}, H_{-23}$ and $H_{23},$ and so on. These modes have same propagation constant (mode matching or perfect phase matching) at the crossing point. Generally, mode matching can be realized in anisotropic waveguides and can be applied for efficient fiber-to-planar waveguide coupling, mode filtering, and improvement of mode conversion efficiency\(^{[25]}\). Mode intersection also occurs in conventional chiral fibers\(^{[23]}\) and negative refractive index fibers\(^{[25]}\).

4) Surface modes exist for $m = -1, 1, -2, 2$. Surface modes connect continuously to the first guided modes at the crossing point of dashdotted (corresponding to $n_{1+}$) and solid or dashed curves. The slopes of dispersion curves of surface modes are negative (i.e., the normalized propagation constant $\beta/k_0$ increases monotonically as the normalized frequency $V$ decreases). The slope of the curves of $H_{11}$ and $H_{21}$ guided modes and $H_{1s}, H_{2s}$ surface modes are very steep, which may have potential application in high-sensitivity optical sensor in the future. Surface modes have been previously found in fiber\(^{[25,26]}\), as well as rectangular\(^{[27]}\) and slab\(^{[28]}\) waveguides, filled with negative index materials.

In the conventional chiral fiber, the energy flux $S_z$ in the cladding is positive and decays exponentially from the interface between the core and cladding. However, there are exotic features for $H_{11}$ and $H_{1s}$ modes (and $H_{21}, H_{2s}$ modes) in the chiral negative refractive index fiber. The energy flux $S_z$ is negative in the cladding and positive in the core. This phenomenon has also been found in chiral nihility fiber\(^{[21]}\); however, it has not been found in
The distributions of energy flux and other higher-order guided modes for a fixed normalized frequency $V$ at $\beta/k_0$ are shown in Fig. 4. For a larger propagation constant $\beta/k_0=1.4735$, the energy flux within the negative refractive index region has also been found in the chiral nihility fiber$^{[26]}$. For a middle propagation constant $\beta/k_0=1.3337$ (Fig. 4(c)), the energy flux $S_2$ is always negative in the core. The normalized power is positive for larger and smaller propagation constants, and negative for a middle propagation constant.

The characteristics of normalized power $P$ for $H_{-12}$, $H_{-13}$, and other higher-order guided modes are similar. Figure 5(a) shows the normalized power $P$ versus normalized frequency $V$ for $H_{-14}$ guided mode. The dispersion curve is also plotted in Fig. 5(b) in order to demonstrate clearly. For larger $V$, the normalized power is equal to 1 (region $E_1D_1$ or $ED$). $P$ decreases from 1 to 0 as $V$ decreases to the lowest limiting frequency, which corresponds to actual cutoff of $H_{-14}$ guided mode (region $E_1C_1$ or $DC$). In the region of negative slope of the dispersion curve (region $C_1B_1$ or $CB$), $P$ is negative. Then, $P$ increases to 1 in the region of positive slope of dispersion curve at smaller propagation constant (region $B_1A_1$ or $BA$). Interestingly, the power $P$ at points $B_1$ and $C_1$ is equal to zero, corresponding to zero group velocity. This implies that at these points, the waveguide cannot propagate energy, i.e., it would be able to halt the light. Waves with this feature are of great practical interest for optical communication and data storage applications.

Now we consider the second case: $n_1 < n_2$. If we choose $\varepsilon_1 = 0.09 \varepsilon_0$, $\mu_1 = \mu_0$, $\varepsilon_2 = \varepsilon_0$, $\mu_2 = \mu_0$, and $k = 2.0$, then refractive index parameters $n_{1\pm} = 2.3$, $n_{1-} = -1.7$, $n_1 = 0.3$, and $n_2 = 1.0$, which satisfy $n_{1-} < 0$ (chiral negative refractive index), $n_1 < n_2$, and $n_2 < |n_{1-}| < n_{1+}$. 

Fig. 3. Energy flux $S_2$ for $H_{11}$ and $H_{-11}$ guided modes. (a) $H_{11}$ mode, $V=0.05$, $\beta/k_0=2.2606$; (b) $H_{-11}$ mode, $V=0.35$, $\beta/k_0=1.4735$.

Fig. 4. Energy flux $S_2$ of three propagation constants at $V=1.25$ for $H_{-12}$ guided mode. (a) $\beta/k_0=3.4638$; (b) $\beta/k_0=1.0487$; (c) $\beta/k_0=1.3337$.

Fig. 5. (a) Normalized power $P$ versus normalized frequency $V$ for $H_{-14}$ guided mode; (b) dispersion curve of $H_{-14}$ guided mode.

Fig. 6. Dispersion curves of modes in the chiral negative refractive index fiber for $n_1 < n_2$. (a) $m=0$; (b) $m=-1,1$; (c) $m=-2,2$. 

The chiral metamaterial slab$^{[14]}$, grounded slab$^{[15,16]}$, and negative refractive index fibers$^{[25]}$. Figure 3(a) shows the energy flux $S_2$ at normalized frequency $V=0.05$ for $H_{11}$ guided mode. The energy flux $S_2$ is almost a constant in the core. For comparison, Fig. 3(b) shows the energy flux $S_2$ at normalized frequency $V=0.35$ for $H_{-11}$ guided mode. In both Figs. 3(a) and (b), the energy flux is in opposite directions in the core and cladding. The normalized power is negative at $V=0.05$ for $H_{11}$ guided mode and $V=0.35$ for $H_{-11}$ guided mode; therefore, they are backward waves. Furthermore, the normalized powers of all surface and guided modes are always negative in the region of negative slope of dispersion curves, as shown in Fig. 2.

There are three propagation constants for $H_{-12}$, $H_{-13}$, and other higher-order guided modes for a fixed $V$. The distributions of energy flux $S_2$ are different for different propagation constants. The energy flux $S_2$ at normalized frequency $V=1.25$ for $H_{-12}$ guided mode is shown in Fig. 4. For a larger propagation constant $\beta/k_0=3.4638$ (Fig. 4(a)) and a smaller propagation constant $\beta/k_0=1.0487$ (Fig. 4(b)), the energy flux $S_2$ is negative near the interface between the core and cladding, and positive near the center of the core. Change in sign of the energy flux within the negative refractive index region has also been found in the chiral nihility fiber$^{[23]}$ and
Although \( n_1 < n_2 \), guided modes can exist in the chiral negative refractive index fiber because the chiral negative refractive index fiber core behaves as a combined medium of two media with effective index \( n_{1+} = 2.3 \), and \( n_{1-} = -1.7 \), respectively. The absolute values of effective index are greater than 1; thus, total internal reflection can occur at the interface between the chiral negative refractive index fiber core and air cladding. Figure 6 shows the normalized propagation constant \( \beta/k_0 \) versus normalized frequency \( V \) for modes in the chiral negative refractive index fiber for \( n_1 < n_2 \). Apart from mode bifurcation and intersection, “S” shape dispersion curve, and surface modes, as discussed in the case of \( n_1 > n_2 \), some other abnormal characteristics can be found from Fig. 6.

1) Shapes of dispersion curves of \( H_{01} \) and \( H_{11} \) guided modes look similar to the letter “L”.

2) \( H_{02} \), \( H_{12} \), and \( H_{-12} \) guided modes have two cutoff frequencies (i.e., these guided modes exist in the region between two normalized cutoff frequencies). This phenomenon has not been found in negative refractive index fibers[25].

3) The shape of dispersion curve of \( H_{04} \) guided mode is elliptical.

Figure 7 shows the energy flux \( S_z \) at different normalized frequency \( V \) for \( H_{1s} \) and \( H_{11} \) modes. For \( H_{1s} \) surface mode, at \( V=0.7 \), the energy flux \( S_z \) in the core is positive, while the energy flux \( S_z \) in the cladding is negative near the interface and positive, away from the interface. For \( H_{1s} \) surface mode, at \( V=0.8 \), the energy flux \( S_z \) in the core is positive near the center of the core and negative near the interface, while the energy flux \( S_z \) in the cladding is positive. For \( H_{11} \) guided mode, at \( V=1.5 \), the energy flux \( S_z \) in the core is positive near the center of the core and negative near the interface, and positive in the cladding. However, the absolute value of energy flux \( S_z \) is the largest at the interface. For all these cases, the normalized power is positive, and the modes are forward waves.

Figure 8 shows the normalized power \( P \) versus the normalized frequency \( V \) for \( H_{04} \) guided mode; dispersion curve is also plotted. In the regions of \( AB \) and \( CD \) in the dispersion curve, \( P \) is positive (regions \( A_1B_1 \) and \( C_1D_1 \)). In the region of \( BC \) in dispersion curve, \( P \) is negative (region \( B_1C_1 \)). In addition, the power \( P \) at points \( B_1 \) and \( C_1 \) is equal to zero, corresponding to zero group velocity.

The abnormal results for two cases discussed above stem from the behavior of the chiral negative refractive index fiber core as a combined medium of two media with effective index \( n_{1+} \) (negative index material) and \( n_{1+} \) (conventional material). In the negative index material, the wave vector propagation is anti-parallel with the energy flux; in the conventional material, the wavevector propagation is parallel with the energy flux. In addition, the absolute value of \( n_{1+} \) is larger than the refractive index of cladding. As a result, complex characteristics of guided mode in the chiral negative refractive index fiber emerge.

The characteristics of guided modes in the chiral negative refractive index fiber that consists of a chiral metamaterial core with an achiral cladding have been investigated theoretically in this letter. We obtain the characteristic equation of guided modes. We plot and present the dispersion curves, energy flux, and power of several low-order guided modes for two type chiral metamaterials parameters. Some novel features are found in the chiral negative refractive index fiber, such as the existence of surface mode and dispersion curves with different abnormal shape, intersection of dispersion curves of different guided modes, negative energy flux in the achiral cladding, and zero power at some normalized frequencies. Some of these characteristics have also been found in the negative refractive index fiber. However, some characteristics, such as negative energy flux in the achiral cladding, mode bifurcation, and guided modes that exist in the region between two normalized cutoff frequencies, have not been found in the negative refractive index fiber. The results presented here will be helpful for potential applications in new fiber devices.

This work was supported by the Natural Science Foundation of Zhejiang Province (No. Y1091139) and partially sponsored by the K. C. Wong Magna Fund in Ningbo University.
References