Research on Volume Scattering Phase Function under Ultraviolet Non-Line-of-Sight Single Scattering Link

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Abstract Based on ultraviolet single scattering link communication, the scattering particle is regarded as a differential volume, whose phase function after scattering is integrated under the correct scattering angles on the different positions within the effective scattering volume. Furthermore, the process is analyzed and formulas are deduced, which provide good support and solid foundation for ultraviolet communications.

Key words optical communication; ultraviolet communication; non-line-of-sight; single scattering; phase function

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1 Introduction

The aim of any communication system is transferring information from one location to another. Information shift is almost completed through modulating information to electromagnetic carrier frequency. Modulated carrier is then transmitted to the destination and demodulated after received. In radio system, carrier is selected from the radio frequency. Microwave and millimeter wave are selected as carriers in the microwave and millimeter communication system. Optical frequencies are chosen in optical communication, such as infrared, visible and ultraviolet (UV) [1]. Among these communications, optical communication has superiority due to the broad unlicensed frequency and wide bandwidth.

Infrared and ultraviolet are both good carriers in optical communication [2]. Infrared can work only when transmitter correctly faces to the receiver. But this term can not be satisfied in harsh conditions [3]. Since the wavelength of ultraviolet is shorter than infrared, ultraviolet is scattered more strongly than infrared by constituents in atmosphere. Thus, ultraviolet taken as carrier can achieve non-line-of-sight (NLOS) communication, which has wider application than infrared communication requiring strict pointing between transmitter and receiver.

This paper mainly studies the scattering phase function, which shows the different scattering intensity with the change of scattering angle. The scattering particle in the common volume is seen as a differential volume and the scattering phase function can be achieved through integrating under the correct scattering angles on the different positions within the effective scattering volume.

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2 Ultraviolet single scattering link model

The model is shown in Fig.1, in which $T$ is transmitter position, $R$ is receiver position, $\theta_i$ and $\varphi_i$ are transmitter apex and divergence, $\theta_r$ and $\varphi_r$ are receiver apex and field of view (FOV), $V$ is overlapping volume, $r$ is distance between $T$ and $R, r_i$ is distance between $T$ and $V, r_o$ is distance between $V$ and $R$, scattering angle $\theta = \theta_i + \theta_r$. Ultraviolet is transmitted with apex and divergence as $\theta_i$ and $\varphi_i$ at $t = 0$. When $t = r/c$, ultraviolet reach overlapping volume $V$ which is full of kinds of particles. These particles scatter ultraviolet to every direction. Those dropping in receiving range can be received by destination receiver at $t = (r_1 + r_o)/c$.

![Fig.1 Single scattering link model of NLOS UV communication](image)

The whole communication can be seen as NLOS process, which can also be seen as two line-of-sight processes, like $T \to V$ then $V \to R$. According to Ref.[4], receiving power at receiver is defined as

$$
P = \frac{P_t}{\Omega_t} \left( \frac{r_i}{r_i} \right)^2 \lambda^2 \left( \frac{\lambda}{4\pi r_o} \right)^2 \exp(-K_r r_o) 4\pi A_r \frac{4\pi A_i}{\lambda^2},
$$

where $P_t$ is transmitting power, $\Omega_t$ is transmitting solid angle, $K_r$ is extinction coefficient, $A_r$ is receiving aperture, $\lambda$ is wavelength, $P(\theta)$ is scattering phase function. From Eq.(1) it’s found that receiving power depends on the same scattering angle $\theta$ wherever the particle lies within the overlapping volume.

3 Volume scattering phase in ultraviolet communication

Ultraviolet scattering is mainly divided into two kinds. When particle size is shorter than UV wavelength, Rayleigh scattering happens. When particle size is longer than UV wavelength, Mie scattering happens. Rayleigh scattering is diffusion scattering, which characterizes atmosphere channel as quiet and clean one. However, Mie scattering is raised by big particles, i.e., fog, smoke, and little globule, which is even severe. Therefore, study of Mie scattering is of important significance for ultraviolet communication.

3.1 Mie scattering phase function

The ratio of scattering energy on a given direction in unit solid angle to average energy on all directions in unit solid angle is called scattering phase function. Scattering phase function is an important parameter in particle scattering character, which reflects scattering ability of atmospheric particles for different scattering angles between $0^\circ$ and $180^\circ$.

Mie scattering process is very complex. Except for traditional Mie theory, empirical Mie phase function, such as Henyey–Greenstein function, revised Henyey–Greenstein function, and Henyey–Greenstein function defined by Cronette and Shanks are used in analoging atmospheric transmission.[6–7]

Henyey–Greenstein phase function (H–G) is defined as

$$
p_{HG}(\theta,g) = \frac{1 - g^2}{(1 + g^2 - 2g \cos \theta)^{\frac{3}{2}}},
$$

Henyey–Greenstein function (H–G1) is defined by Cronette and Shanks as

$$
p_{HG1}(\theta,g) = \frac{3}{2} \times \frac{1 - g^2}{2 + g^2} \frac{1 + \cos \theta}{(1 + g^2 - 2g \cos \theta)^{\frac{3}{2}}}.
$$

Revised Henyey–Greenstein function (H–G2) is shown as

$$
p_{HG2}(\theta,g) = \frac{1 - g^2}{4\pi} \left[ \frac{1}{(1 + g^2 - 2g \cos \theta)^{\frac{3}{2}}} + f \frac{0.5(3 \cos^2 \theta - 1)}{(1 + g^2)^{\frac{3}{2}}} \right],
$$

where $g$ is asymmetry faceter and $f$ is scattering faceter.
Seen from Eqs.(2)–(4), change of $\theta$ will change phase function no matter which function is employed to compute phase function.

### 3.2 Scattering phase function in UV communication

In section 2, the receiving power is defined as Eq.(1). Seen from Eq.(1), the overlapping volume can be imagined as countless differential volume $dv$. Each $dv$ takes responsibility of scattering ultraviolet. Receiving power is computed through integrating effect of all $dv$.

Each $dv$ is taken as a scattering particle. Due to each $dv$ at different location within overlapping volume, the apex angles from $T$ to $dv$ and $dv$ to $R$ for each $dv$ are different, which is to say, for each $dv$ on different location, $\theta_t$ and $\theta_r$ are different from others. After scattering incident wave, all scattering angles which can drop in receiving range for each $dv$ are $\theta = \theta_t + \theta_r$. Thus, the scattering angle for each $dv$ is different. $\varphi_t$ is transmitting divergence angle associated with scattering volume, $\varphi_r$ is receiving divergence angle associated with scattering volume, $\theta_t$ is the angle between baseline and line of scattering volume and transmitter, $\theta_r$ is the angle between baseline and line of scattering volume and receiver, $\xi_{\text{min}}$ and $\xi_{\text{max}}$ are the maximum and minimum limits of $X$ axis. As shown in Fig.2, for example, three $dv$ on different positions have different scattering angles $\theta_1$, $\theta_2$, $\theta_3$. Due to $r > D^2/\lambda$ ($D$ is the size of scattering particle), scattering of each particle in the overlapping volume is irrelevant. Therefore, we can add the contribution of every particle directly. Based on this condition, we should integrate the different phase functions raised by each $dv$.

![Fig.2 Different scattering angles within receiving range in the effective scattering volume](image1)

### 3.3 Volume scattering phase function based on prolate–spheroid coordinate

Prolate–spheroid coordinate is shown in Fig.3, in which ellipse is rotated around the main axis to complete a prolate–spheroid surface. Arbitrary point on prolate–spheroid is determined with a radial coordinate $\xi$, an angular coordinate $\eta$ and an azimuthal coordinate $\varphi$ \cite{8}. Fig.4 is the application of prolate–spheroid coordinate in UV single scattering link model. Integration of phase function $P(\theta)$ is to integrate $\xi$, $\eta$, $\varphi$ within overlapping volume, which can be defined as

$$\int_{\xi_{\text{min}}}^{\xi_{\text{max}}} d\xi \int_{\eta_{\text{min}}}^{\eta_{\text{max}}} P(\cos \theta) d\varphi,$$

(5)

![Fig.3 Prolate–spheroid coordinate](image2)
where scattering angle \( \theta = \psi_1 + \psi_2 \). When parameters relevant to \( \psi_1 \) and \( \psi_2 \) change, phase function will change. Three ranges of \( \xi, \eta, \varphi \) are discussed as follows.

### 3.3.1 Radial coordinate

In Fig. 3, \( \eta_i \) is a function related to \( \xi \) and \( \psi_i \) and \( \eta_i \) is a function related to \( \xi \) and \( \psi_i \) \( (i=1, 2) \), \( F_1 \) is the transmitter, \( F_2 \) is the receiver, \( \psi_i \) is transmitting angle, and \( \psi_i \) is receiving angle. There are functions shown as below:

\[
\cos(\psi_1) = \frac{1 + \xi \eta}{(\xi + \eta)},
\]

\[
\cos(\psi_2) = \frac{1 - \xi \eta}{(\xi - \eta)}.
\]

According to Eqs. (6) and (7),

\[
\eta_1 = \frac{\xi \cos(\psi_1) - 1 - \xi \cos(\theta_i)}{\xi - \cos(\theta_i)},
\]

\[
\eta_2 = 1 - \xi \cos(\psi_1) = \frac{1 - \xi \cos(\theta_i)}{\xi - \cos(\theta_i)} ,
\]

where \( \psi_i = \theta_i, \psi_i = \theta_i \). According to Eqs. (8) and (9), change of \( \eta_i \) and \( \eta_2 \) is mainly relevant to \( \xi \).

![Fig. 4 UV link under prolate-spheroid coordinate](image)

### 3.3.2 Angular coordinate

When \( \xi \) adopt minimum \( \xi_{\min} \), \( \eta_i = \eta_2 \). Make Eq. (8) equivalent to Eq. (9), where \( \psi_i = \theta_i - \varphi_i, \psi_2 = \theta_i - \varphi_i \), and \( \xi_{\min} \) is defined as

\[
\xi_{\min} = \frac{1 + \cos(\psi_1) \cos(\psi_2)}{\cos(\psi_1) + \cos(\psi_2)} + \sqrt{\frac{1 + \cos(\psi_1) \cos(\psi_2)}{\cos(\psi_1) + \cos(\psi_2)}}^2 - 1 = \frac{1 + \cos(\theta_i - \varphi_i) \cos(\theta_i - \varphi_i)}{\cos(\theta_i - \varphi_i) + \cos(\theta_i - \varphi_i)} + \sqrt{\frac{1 + \cos(\theta_i - \varphi_i) \cos(\theta_i - \varphi_i)}{\cos(\theta_i - \varphi_i) + \cos(\theta_i - \varphi_i)}}^2 - 1.
\]

when \( \xi \) adopts maximum \( \xi_{\max} \), \( \eta_i = \eta_2 \). Make Eq. (8) equivalent to Eq. (9), where \( \psi_i = \theta_i + \varphi_i \), \( \psi_2 = \theta_i + \varphi_i \), and \( \xi_{\max} \) is defined as

\[
\xi_{\max} = \frac{1 + \cos(\psi_1) \cos(\psi_2)}{\cos(\psi_1) + \cos(\psi_2)} + \sqrt{\frac{1 + \cos(\psi_1) \cos(\psi_2)}{\cos(\psi_1) + \cos(\psi_2)}}^2 - 1 = \frac{1 + \cos(\theta_i + \varphi_i) \cos(\theta_i + \varphi_i)}{\cos(\theta_i + \varphi_i) + \cos(\theta_i + \varphi_i)} + \sqrt{\frac{1 + \cos(\theta_i + \varphi_i) \cos(\theta_i + \varphi_i)}{\cos(\theta_i + \varphi_i) + \cos(\theta_i + \varphi_i)}}^2 - 1.
\]

According to Eqs. (10) and (11), scattering angle mainly depends on changing \( \xi \), which embodies the change of \( \psi_1 + \psi_2 \) from \( \phi_1 = \theta_i - \varphi_i, \phi_2 = \theta_i - \varphi_i \) to \( \phi_1 = \theta_i + \varphi_i, \phi_2 = \theta_i + \varphi_i \). At the same time, \( \eta_i \) and \( \eta_2 \) are also interfered.

### 3.3.3 \( \phi_1 \) and \( \phi_2 \)

Transmitting cone of light is of symmetry with receiving cone according to \( X-Z \) plane, thus, the overlapping volume is of symmetry according to \( X-Z \) plane.

\[
\phi_1 = -\phi_1,
\]

\[
\phi_2 = \arctan[r_1 \sin(\theta_i), r_1 \tan(\varphi_i)] = -\phi_1.
\]
4 Conclusion

Aiming at phase function at different position within overlapping volume, phase function with different scattering angles dropping in receiving range is integrated, and integration limit is analyzed and deduced, which gives receiving power more correct quantity.

参考文献