Graphene-based surface plasmon waveguides (SPWs) show high confinement well beyond the diffraction limit at terahertz frequencies. By combining a graphene SPW and nonlinear material, we propose a novel graphene/AlGaAs SPW structure for terahertz wave difference frequency generation (DFG) under near-infrared pumps. The composite waveguide, which supports single-mode operation at terahertz frequencies and guides two pumps by a high-index-contrast AlGaAs/AlOₓ structure, can confine terahertz waves tightly and realize good mode field overlap of three waves. The phase-matching condition is satisfied via artificial birefringence in an AlGaAs/AlOₓ waveguide together with the tunability of graphene, and the phase-matching terahertz wave frequency varies from 4 to 7 THz when the Fermi energy level of graphene changes from 0.848 to 2.456 eV. Based on the coupled-mode theory, we investigate the power-normalized conversion efficiency for the tunable terahertz wave DFG process by using the finite difference method under continuous wave pumps, where the tunable bandwidth can reach 2 THz with considerable conversion efficiency. To exploit the high peak powers of pulses, we also discuss optical pulse evolutions for pulse-pumped terahertz wave DFG processes.

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OCIS codes: (190.4223) Nonlinear wave mixing; (240.6680) Surface plasmons; (230.7370) Waveguides; (160.6000) Semiconductor materials.

https://doi.org/10.1364/PRJ.6.000186

1. INTRODUCTION

Graphene has recently attracted huge interest in the optoelectronics field [1], such as passively mode-locked lasers [2,3], broadband polarizers [4], chemical gas sensing [5], optical modulators or switches [6,7], photodetectors [8], enhanced four-wave mixing [9], terahertz (THz) and infrared spectroscopy [10], and plasmonics [11,12]. Graphene-based plasmons enable strong confinement of the optical field at subwavelength scales and hold great potential for applications [13], which can be tuned via gate bias voltage, chemical doping, the electric field, or the magnetic field [14]. To guide graphene surface plasmons with deep subwavelengths, many types of graphene surface plasmon waveguides (GSPWs) have been proposed [4,6,9,15,16]. At THz frequencies, the tunability, excitation, and generation of graphene surface plasmons have been demonstrated [17–19]. Liu et al. investigated the effective refractive index, loss, and effective mode area of a 100 μm width Si–SiO₂–graphene–dielectrics–graphene–SiO₂–Si waveguide for different Fermi energy levels (chemical potentials) [20], where the loss and effective index can be reduced with increasing Fermi energy level when the resultant optical mode area increases. Zhou et al. proposed a hybrid few-layer GSPW to balance the loss and mode area through the strong coupling between the dielectric waveguide and plasmon waveguide [21]. Furthermore, Xu et al. analyzed the single-mode operation region of a dielectric-loaded GSPW with different Fermi energy levels [22]; the optical properties of the fundamental mode can be tuned by changing the Fermi energy level, and the waveguide is simple and easy for fabrication.

THz sources are important for many applications of THz waves [23]. The four-wave mixing (FWM) and difference frequency generation (DFG) in waveguide devices are important techniques to generate coherent THz waves through nonlinear wavelength conversion. Barh et al. reported an all-fiber FWM THz source in a microstructured-core double-clad plastic fiber pumped by a high-power CO₂ laser [24]. Sun et al. proposed a hybrid graphene sheets waveguide for THz FWM generation [25], where the waveguide supports graphene surface plasmon modes at THz frequencies. Compared with the FWM process, the DFG technique is more attractive because of its high
quadratic nonlinearity, which has been investigated in many conventional waveguides, such as the LiNbO₃ ribbon waveguide [26], the periodically poled LiNbO₃ rectangular waveguide [27], the THz surface-emitted GaAs/AlAs modal birefringence waveguide [28], and the Al₀.₅Ga₀.₅As nested waveguide heterostructure [29]. Due to the large wavelength difference between the optical pump and THz waves, the THz conventional waveguides have a dimension of hundreds of microns, so the mode field overlaps are unsatisfactory. To confine THz more tightly, we proposed an AlGaAs photonic crystal waveguide with the scales of tens of microns to achieve THz generation [30]. Ruan et al. analyzed the THz DFG from a micron-scale GaAs strip, which is embedded within a metallic slot waveguide [31], but the THz mode field concentrates in the metallic region and leads to a reduced mode field overlap. Ge et al. investigated a gold nanowire array on the surface of lithium niobite crystal for guiding both optical and terahertz waves by surface plasmonic modes with a rather small mode overlapping area [32], and the THz waves can be surface-emitted based on the Cherenkov phase-matched DFG process with 4 μm propagation length, which is smaller than the coherence length. On the other hand, the conversion bandwidth of the THz DFG process is quite narrow in conventional waveguides because of the strict phase-matching condition.

To exploit the valuable deep-subwavelength confinement and tunability of the THz GSPW, in this paper we analyze in detail the use of a graphene/AlGaAs surface plasmon waveguide for colinear and tunable THz wave DFG under near-infrared pumps. At THz frequencies, the waveguide provides single-mode surface plasmon confinement with highly effective refractive indices, and the mode field overlap between pump and THz waves is greatly improved. The phase-matching condition is achieved by adjusting the Fermi energy level of graphene and employing the artificial birefringence at near-infrared frequencies in high-index-contrast AlGaAs/AlOₓ waveguides. We mainly focus on THz wave generation with a 6 THz central frequency; tunable THz wave generation from 4 to 7 THz is realized through changing the signal wavelength and adjusting the Fermi energy level of graphene from 0.848 to 2.456 eV. Based on coupled-mode theory, we investigate the conversion efficiency and pulse evolutions of tunable THz wave DFG processes by using the finite difference method under continuous wave (CW) and pulse pumps, respectively. Such a tunable THz source, which can allow larger manufacturing tolerances, may be easy to integrate with other THz GSPW structures for applications.

2. Waveguide Structure and Theoretical Model

The waveguide structure, as shown in Fig. 1, consists of a nonlinear Al₀.₅Ga₀.₅As strip embedded in an AlOₓ-loaded GSPW. The Al₀.₅Ga₀.₅As core layer is chosen to avoid two-photon absorption and for its easy fabrication process [33,34]. Owing to a negligible difference of refractive indices between Al₀.₅Ga₀.₅As and AlOₓ at THz frequencies [35], the mode field will concentrate in the central Al₀.₅Ga₀.₅As region. The high-index-contrast AlGaAs/AlOₓ structure is designed to provide strong mode field confinement and birefringence between TE and TM modes for two pumps, which is important for good mode field overlap and the phase-matching condition. W, H, w, and b are the width and height of AlOₓ and Al₀.₅Ga₀.₅As strips, respectively. The structure may grow on GaAs substrates (001) crystal direction by using standard semiconductor growth and processing techniques [e.g., molecular beam epitaxy or plasma-enhanced chemical vapor deposition (PECVD)] [36], where Al₀.₉₃Ga₀.₀₇As is used for the lower cladding layer. Photolithography, regrows Al₀.₅Ga₀.₅As and Al₀.₉₃Ga₀.₀₇As to obtain the lower waveguide structure, and then thermally oxidation converts the Al₀.₉₃Ga₀.₀₇As lower cladding layers to AlOₓ layers [37]. The monolayer graphene may be fabricated by high-quality CVD, which grows on the polished lower waveguide structure [38], and the upper AlOₓ strip may be formed by the atomic layer deposition technique [6,39].

The model considers the THz DFG process, and the propagation direction is along the x axis [(110) crystal direction, as shown in Fig. 1. Under the slowly varying envelope approximation, where the electric field amplitude changes slowly relative to the fast optical carrier frequency, we can derive the following coupled-mode equations for the DFG process [40]:

\[
\begin{align*}
\frac{\partial A_p}{\partial t} + \beta_p \frac{\partial A_p}{\partial x} + \frac{1}{2} \alpha_p A_p &= \omega_p \kappa A_s A_i \exp(-i\Delta k x), \\
\frac{\partial A_s}{\partial t} + \beta_s \frac{\partial A_s}{\partial x} + \frac{1}{2} \alpha_s A_s &= \omega_s \kappa A_p A_i \exp(i\Delta k x), \\
\frac{\partial A_i}{\partial t} + \beta_i \frac{\partial A_i}{\partial x} + \frac{1}{2} \alpha_i A_i &= \omega_i \kappa A_p A_s \exp(i\Delta k x),
\end{align*}
\]

where \(A_{p,i}\) are the normalized complex amplitudes of the pump, signal, and THz waves, respectively, which are as functions of the position \(x\) and time \(t\). \(\left|A_{p,i}\right|^2 = P_{p,i}\) denote the powers. \(\beta_{p,i}\) stand for the reciprocal value of group velocity, and \(\alpha_{p,i}\) are the propagation loss coefficients. \(\omega_{p,i}\) are the angular frequencies, and \(\omega_i = \omega_p - \omega_s\) satisfies energy conservation. \(\Delta k = k_p - k_i - k_l\) expresses the phase mismatch, and \(k_{p,i}\) are the corresponding wavevectors for \(\omega_{p,i}\). The parameter \(\kappa\) refers to the DFG coupling coefficient,

\[
\kappa = d_{eff} \sqrt{2\mu_0/(c\eta_p\eta_s\eta_iA_{eff})},
\]
where $d_{\text{eff}}$ is the effective nonlinear coefficient. For the DFG process in the GSPW of Fig. 1, the TE pump ($\omega_p$) and TM signal ($\omega_s$) generate TM THz wave ($\omega_i$). Thus, $d_{\text{eff}} = d_1/4 = 47 \text{ pm/V}$, where $d_1$ is the AlGaAs nonlinear coefficient [29]. $n_{\text{p,i}}$ are the effective refractive indices. $A_{\text{eff}}$ is the effective nonlinear interaction area and can be expressed as

$$A_{\text{eff}} = \frac{\iint |\mathbf{E}|^2 \, dz \, dz \int |\mathbf{E}|^2 \, dy \, dz}{\iint \mathbf{E} \varepsilon_j \mathbf{E} \, dz \, dz \int \mathbf{E} \varepsilon_j \mathbf{E} \, dy \, dz}, \quad (3)$$

where $\varepsilon_{p,i}$ are the transverse mode field profiles.

### A. Field Distribution

The relative permittivity of graphene is equivalently expressed as $\varepsilon_g = 1 + \frac{\alpha_{\text{rel}}}{\omega^2}$, and monolayer graphene can be treated as an ultrathin film layer with a thickness of $\delta = 0.5 \text{ nm}$ [14], $\omega$ is the angular frequency of optical waves. $\delta_1$ is the optical surface conductivity of graphene, which is related to temperature $T$, angular frequency $\omega$, Fermi energy level $E_F$, and momentum relaxation time $\tau$ [41]. We have

$$\delta_1 = i \left( \frac{e^2}{\pi \hbar^2 (\omega + i \tau^{-1})} \varepsilon_g \right) \left( \frac{E_F}{k_B T} \right) + 2 \ln \left[ \exp \left( -\frac{E_F}{k_B T} \right) + 1 \right]$$

$$+ i \frac{e^2}{4\pi \hbar} \left[ 2 |E_F| - \hbar (\omega + i \tau^{-1}) \right]$$

$$+ \frac{e^2}{4\pi \hbar} \left[ 2 |E_F| + \hbar (\omega + i \tau^{-1}) \right], \quad (4)$$

where $k_B$ is the Boltzmann constant, and $T = 300 \text{ K}$ at room temperature. Relaxation time $\tau = \mu E_F/(e \varepsilon_g)$ is determined by the carrier mobility $\mu$, Fermi velocity $v_F$, and Fermi energy level $E_F$. The parameters $\mu$ and $v_F$ are considered to be $10,000 \text{ cm}^2/(\text{V} \cdot \text{s})$ and $10^8 \text{ m/s}$, respectively [22].

The refractive index of graphene is $\sqrt{\varepsilon_g} = n + ik$, where $n$ and $k$ correspond to the real and imaginary parts of the refractive index, as shown in Figs. 2(a) and 2(b), respectively. $n$ and $k$ both decrease with increasing THz wave frequency when $k$ increases with the increasing $E_F$. We calculate the effective refractive indices and mode field distribution of three waves by the finite-element-method-based commercial COMSOL mode solver. We analyze the complex effective refractive indices $n_{\text{eff}}$ for single-mode THz GSPW under different values of $n$ and $k$. The real part of $n_{\text{eff}}$ is $n$, in Eq. (4), and the imaginary part of $n_{\text{eff}}$ indicates the loss as $\alpha_i = 4\pi \text{ Im}(n_{\text{eff}})/\lambda_i$ in Eq. (1). $W$ and $H$ in Fig. 1 are adjusted to realize the single-mode operation. It is found that $n$ affects the loss $\alpha_i$ and $k$ relates to the mode field confine, respectively. When $E_F$ increases, the smaller $n$ brings about lower loss and the bigger $k$ requires larger dimension $W$ and $H$ for single-mode operation. Therefore, there is a tradeoff between loss and mode field confinement. That is, while the loss is reduced, the single-mode structure will result in a large mode field area. It is worth noting that we also investigate the influence of few-layer graphene (e.g., two layers) on the THz GSPW, where the few-layer graphene just reduces $E_F$ for the single-mode condition with fixed $W$ and $H$, but the loss cannot be reduced.

Figures 3(a)–3(c) show the electric field of the TM single mode for 6 THz and fundamental modes for $\lambda_p = 1.53 \mu m$ and $\lambda_i = 1.5783 \mu m$ under the phase-matching condition, which will be discussed in the following. From Fig. 3(a), the waveguide confines the THz mode field very tightly and the electric field is restricted around the AlGaAs dielectric. In vertical direction, it is continuous for the electric field of TE $\lambda_p$, as shown in Fig. 3(b), or for the electric displacement field of TM $\lambda_i$, as shown in Fig. 3(c). Calculating the overlap integral of Eq. (3), we obtain the effective nonlinear interaction
can be accurately derived from the formula for our case. We fix $s$ to satisfy the phase-matching condition by changing the Fermi energy level [6,42], and the absorption is very little treated as a tunable absorption layer by electrically tuning to be 0.2. At the near-infrared regime, the graphene can be treated as a tunable absorption layer by electrically tuning [43]. In detail, the effective refractive index of graphene is estimated $0.2$. At the near-infrared regime, the graphene can be treated as a tunable absorption layer by electrically tuning the Fermi energy level [6,42], and the absorption is very little for our case. We fix $w = 0.8 \text{ m}$ and $b = 0.4 \text{ m}$ and try to satisfy the phase-matching condition by changing the Fermi energy level $E_F$.

Figure 4(a) plots the phase mismatch for THz generation with different frequencies when the pump wavelength is fixed at $\lambda_p = 1.53 \text{ m}$. By adjusting $E_F$ and signal wavelength $\lambda_s$, the phase-matching condition can be achieved. As shown in Fig. 4(b), when the THz frequency is varied from 4 to 7 THz, the corresponding $E_F$ for phase matching is changed from 0.848 to 2.456 eV and this $E_F$ may be reached through electrical tuning [43]. In detail, the effective refractive index of GSPW and the Fermi energy level for phase-matching at the THz frequency are summarized in Table 1. When the THz frequency decreases, the real part $n_r$ and the loss $\alpha_t$ will increase.

3. RESULTS AND DISCUSSION

A. CW Pump

We first consider the CW pump ($\frac{\partial n}{\partial t} = 0$). The coupled-mode equations in Eq. (1) are solved numerically by using the finite difference method [44], and the computational step size is changed step by step to the requirements of higher resolution. For the low loss of the high-index-contrast AlGaAs/AlO$_x$ waveguide and graphene absorption [45], we assume $\alpha_p = \alpha_s = 5 \text{ dB/cm}$. In the DFG process, the pump and signal wave powers are much higher than the THz wave power. The power-normalized conversion efficiency, which is defined as $\eta = P_s/(P_pP_s)$, are approximately the same for different pump and signal powers. Figure 5(a) shows the power-normalized conversion efficiency as a function of THz wave frequency and waveguide length. From Fig. 5(a), the conversion efficiency increases first and then decreases slightly along the waveguide. This is because the lost THz wave gradually exceeds the generated. With 500 $\mu\text{m}$ waveguide length, the THz wave frequency can range from 5 to 7 THz when the conversion efficiency is larger than $0.5 \times 10^{-6} \text{ W}^{-1}$; the tunable bandwidth can reach 2 THz. The maximum Fermi energy level $E_F$ restricts the high frequency of the THz wave, as shown in Fig. 5(b), and the large loss limits the low frequency and affects the conversion efficiency.

There are two main mechanisms to suppress the loss. One is enhancing the carrier mobility $\mu$ of graphene [22]. The carrier mobility ranges from 8,000 cm$^2$/(V·s) in mechanical cleavage of bulk graphite [46] to 230,000 cm$^2$/(V·s) in high-quality suspended graphene [47]. We use moderate carrier mobility $\mu = 10,000 \text{ cm}^2/(\text{V} \cdot \text{s})$ in our model. The other one is enlarging the dimension of the waveguide. If width $W$ and height $H$ are of the order of tens of microns, the loss will very small, but meanwhile mode field overlap will be reduced (larger effective nonlinear interaction area $A_{eff}$).

In order to improve THz wave power, the CW pump and signal powers need to be enhanced, which is difficult to achieve in applications. Compared to the CW pump, it is easier to realize high peak power by a pulse pump, which has the requirement

\begin{align}
\Delta k = k_p - k_i - k_s = 0,
\end{align}

where $k_{p;i} = \frac{2\pi n_{p;i}}{\lambda_{p;i}}$ and $\lambda_{p;i} = c/\omega_{p;i}$ are the vacuum wavelengths of three waves. Due to the high THz effective refractive index $n_i$ of GSPW, we employ the birefringence of the high-index-contrast AlGaAs/AlO$_x$ structure to realize the phase matching. Based on $\omega_i = \omega_p - \omega_s$ and the phase-matching condition, the difference of effective refractive indices between pump and signal can be expressed as

\begin{align}
n_p - n_i = \frac{\omega_i}{\omega_p} (n_i - n_i).
\end{align}

The dielectric constant of nonlinear Al$_x$Ga$_{1-x}$As material can be accurately derived from the formula $\varepsilon(\text{Al}_x\text{Ga}_{1-x}\text{As}) = x\varepsilon(\text{AlAs}) + (1 - x)\varepsilon(\text{GaAs})$ [35]. The material absorption factors are associated with the imaginary parts of the refractive index. There are not strict formulas for AlO$_x$ and Al$_x$Ga$_{1-x}$As at THz frequencies, so we obtain the expression by fitting the data from Ref. [35]. We mainly consider the 6 THz wave generation, where the imaginary parts of the refractive index are $1.17 \times 10^{-2}$ and $7.545 \times 10^{-2}$ for revised AlO$_x$ and Al$_x$Ga$_{1-x}$As, respectively. After trying many times, we choose $W = 1.35 \mu\text{m}$ and $H = 2.5 \mu\text{m}$, where $n_s - n_p$ is estimated to be 0.2. At the near-infrared regime, the graphene can be treated as a tunable absorption layer by electrically tuning the Fermi energy level [6,42], and the absorption is very little for our case. We fix $w = 0.8 \mu\text{m}$ and $b = 0.4 \mu\text{m}$ and try to satisfy the phase-matching condition by changing the Fermi energy level $E_F$.

Figure 4(a) plots the phase mismatch for THz generation with different frequencies when the pump wavelength is fixed at $\lambda_p = 1.53 \mu\text{m}$. By adjusting $E_F$ and signal wavelength $\lambda_s$, the phase-matching condition can be achieved. As shown in Fig. 4(b), when the THz frequency is varied from 4 to 7 THz, the corresponding $E_F$ for phase matching is changed from 0.848 to 2.456 eV and this $E_F$ may be reached through electrical tuning [43]. In detail, the effective refractive index of GSPW and the Fermi energy level for phase-matching at the THz frequency are summarized in Table 1. When the THz frequency decreases, the real part $n_r$ and the loss $\alpha_t$ will increase.

![Figure 4](image-url)

**Fig. 4.** (a) Phase mismatch $\Delta k$ as a function of Fermi energy level for 5.8, 5.9, 6.0, and 6.1 THz generation. (b) Phase-matching THz frequency as a function of Fermi energy level.
for the full width at half-maximum (FWHM) of conversion efficiency. Figure 5(b) illustrates the THz wave power for different THz frequencies. The power-normalized conversion efficiency is $\eta = 1.619 \times 10^{-6} \text{W}^{-1}$ for 6 THz and the FWHM is about 0.044 THz in 500 $\mu$m long waveguide.

**B. Pulse Pump**

The pulse width of the transform-limited Gaussian pulse is about 10 ps for 0.044 THz FWHM with 1.53 $\mu$m central wavelength. We use the Gauss pulse pump, and its pulse width is 30 ps. The coupled-mode equations in Eq. (1) are numerically simulated under the walk-off effect, which arises from the very large group index difference between the pump, signal, and THz waves. Figures 6(a)–6(c) display the pulse evolutions for the pump, signal, and THz waves propagating along the waveguide. As shown in Fig. 6(b), the CW signal gradually evolves into a pulsed wave, owing to the DFG process. Figure 6(c) depicts the THz wave evolution, where the peak power rises to a maximum at 500 $\mu$m and then drops slowly. Owing to the short propagation distance, the influence of temporal walk-off between the pump, signal, and THz pulses on the peak power of the THz wave is negligible. Therefore, the power-normalized conversion efficiency of the peak power for

### Table 1. Effective Refractive Index and Fermi Energy Level $E_F$ for Phase-Matching THz Frequency

<table>
<thead>
<tr>
<th>Wave Frequency/THz</th>
<th>4</th>
<th>4.5</th>
<th>5</th>
<th>5.5</th>
<th>6</th>
<th>6.5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_F$/eV</td>
<td>0.848</td>
<td>1.072</td>
<td>1.340</td>
<td>1.606</td>
<td>1.885</td>
<td>2.167</td>
<td>2.456</td>
</tr>
<tr>
<td>Effective index</td>
<td>12.171-0.658i</td>
<td>11.223-0.399i</td>
<td>10.477-0.270i</td>
<td>9.890-0.151i</td>
<td>9.357-0.144i</td>
<td>8.927-0.109i</td>
<td>8.552-0.086i</td>
</tr>
</tbody>
</table>

**Fig. 5.** (a) Contour image of power-normalized conversion efficiency versus waveguide length for different THz wave frequencies. (b) THz wave power versus THz frequency for waveguide length 500 $\mu$m and 2500 $\mu$m with $P_p = 100$ W and $P_s = 1$ W.

**Fig. 6.** Optical pulse evolutions along the waveguide with 30 ps pulse input pump with peak power 100 W and 1 W CW input signal: (a) pump wave power, (b) signal wave power, and (c) THz wave power.
the pulse pump is basically the same as the one for the CW pump, as shown in Fig. 5(a).

4. CONCLUSION

In conclusion, we have investigated in detail a graphene/AlGaAs surface plasmon waveguide for a tunable THz source under near-infrared pumps. To realize the good mode field overlap and phase-matching condition, a hybrid high-index-contrast AlGaAs/Aloₓ and graphene surface plasmon waveguide structure is proposed. The GSPW provides deep-subwavelength confinement and single-mode operation at THz frequencies; the birefringent AlGaAs/Aloₓ can be used to satisfy the phase-matching condition. Owing to the adjustability of the Fermi energy level in graphene, the waveguide can bring about phase-matching THz generation from 4 to 7 THz when the Fermi energy level of graphene is varied from 0.848 to 2.456 eV. Based on the coupled-mode equations, we numerically calculate the DFG process under CW and pulse pumps. Simulation results show that tunable bandwidth can reach 2 THz with considerable power-normalized conversion efficiency. We note that the GSPW provides a new tunable THz source with flexible dimension, which may benefit the integrated application of optical devices.

Funding. National Natural Science Foundation of China (NSFC) (11547187, 11405073, 61405073); Shandong Provincial Key R&D Program (2017CXSQ0416).

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