Supplementary Material

Improving the sensitivity of DC magneto-optical Kerr effect measurement to 10^{-7} rad/$\sqrt{\text{Hz}}$

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1. Jones matrices analysis of balanced detection

In the following, we will analyze the MOKE Kerr rotation using the Jones matrices method[1]. The incoming s-polarized light ($\begin{bmatrix} 0 \\ E_{s0} \end{bmatrix}$), after reflection from a magnetic sample, experiences a small rotation θ_k ($\ll 1$)of the polarization, and the corresponding Jones matrix is given by

$$S = \begin{bmatrix} cos\theta_k & sin\theta_k \\ -sin\theta_k & cos\theta_k \end{bmatrix} \approx \begin{bmatrix} 1 & \theta_k \\ -\theta_k & 1 \end{bmatrix} \tag{1}$$

The second half-wave plate (HWP2) which rotates the polarization direction of light by an angle of $\alpha \to \pi/4$ is described by the matrix

$$R = \begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix} \tag{2}$$

Thus, the optical electric field in the two detection arms after the Wollaston prism reads

$$\begin{bmatrix} E_p \\ E_s \end{bmatrix} = R \times S \times \begin{bmatrix} 0 \\ E_{s0} \end{bmatrix} = \begin{bmatrix} sin\alpha + cos\alpha \cdot \theta_k \\ cos\alpha - sin\alpha \cdot \theta_k \end{bmatrix} \times E_{s0} \tag{3}$$

The intensity difference between the split beams that is sensed by the balanced detector is

$$\Delta I_{signal} = |E_p|^2 - |E_s|^2 \approx (-cos2\alpha + 2\theta_k sin2\alpha)|E_{s0}|^2 \tag{4}$$

In the measurement, the balance between the two arms is carefully adjusted such that the polarization rotation angle α reaches $\pi/4$. Then, the MOKE signal is simply given by

$$\Delta I_{signal} = 2I_0\theta_k \tag{5}$$

Here, I_0 is the intensity of the reflected s-polarized light ($|E_{s0}|^2$).

2. Noise analysis of Laser mode sweeping

The above derivation is valid for an incoming light with pure polarization. However, due to limited extinction ratio of the polarizer, the incident light inevitably consists of large s-component and a residual p-component, which may be expressed as $\begin{bmatrix} E_{p0} \\ E_{s0} \end{bmatrix}$. The extinction ratio ($\beta$) of a commercial birefringence polarizer is usually 10^5, i.e. $\beta = \left|\bar{E}_{s0}/\bar{E}_{p0}\right|^2 \sim 10^5$. Then, Eq. (4) shall be revised as

$$\Delta I_{signal} \approx -cos2\alpha \times |E_{s0}|^2 + 2sin2\alpha \times E_{s0}E_{p0} + 2sin2\alpha \times \theta_k \times |E_{s0}|^2 \tag{6}$$

Considering fluctuations of the incident s- and p-polarized electric field components, they can be expressed as

$$E_{s0} = \bar{E}_{s0} + \Delta E_{s0} \tag{7,a}$$

$$E_{p0} = \bar{E}_{p0} + \Delta E_{p0} \tag{7,b}$$

Here, ΔE_i is the variation of the electric field (E_i) and $\bar{E}_i$ represents the average of E_i. These fluctuations will lead to signal noise in the balance detection with ΔI_{signal} given by

$$\Delta I_{signal} \approx 2|\bar{E}_{s0}|^2\left(\frac{\bar{E}_{p0}}{\bar{E}_{s0}} - \delta\right) + 2\bar{E}_{s0}\bar{E}_{p0}\left[\frac{\Delta E_{p0}}{\bar{E}_{p0}} + \frac{\Delta E_{s0}}{\bar{E}_{s0}}\left(1 - 2\delta\sqrt{\beta}\right)\right] + 2\theta_k|\bar{E}_{s0}|^2\left(1 + 2\frac{\Delta E_{s0}}{\bar{E}_{s0}}\right) \tag{8}$$

In this equation, we have defined $\alpha = \pi/4 - \delta$ with δ being a small quantity. The first term represents the background, the second one describes noise from laser contains and the third is proportional to Kerr rotation.

In an experiment, to evaluate the sensitivity of a MOKE apparatus, one often uses a non-magnetic sample to test the noise background, i.e. setting $\theta_k = 0$. The relative signal difference between the two arms becomes

$$\frac{\Delta I_{signal}}{2I_0} \approx \left(\frac{1}{\sqrt{\beta}} - \delta\right) + \frac{1}{\sqrt{\beta}}\left[\frac{\Delta E_{p0}}{\bar{E}_{p0}} + \frac{\Delta E_{s0}}{\bar{E}_{s0}}\left(1 - 2\delta\sqrt{\beta}\right)\right] \tag{9}$$

Assuming there exists only intensity noise while the polarization remains unchanged, we have $\Delta E_p/\bar{E}_p = \Delta E_s/\bar{E}_s$. Eq. (9) can be simplified into

$$\frac{\Delta I_{signal}}{2I_0} \approx \left(\frac{1}{\sqrt{\beta}} - \delta\right)\left[1 + 2\frac{\Delta E_{s0}}{\bar{E}_{s0}}\right] \tag{10}$$

Clearly, both the signal background and the noise caused by intensity fluctuation is proportional to $\left(1/\sqrt{\beta} - \delta\right)$. In such case, the intensity noise can be suppressed by careful adjustment of the halfwave plate angle. In other words, the common-mode noise from laser intensity can be greatly suppressed by tuning δ approaching $1/\sqrt{\beta}$. However, as evidenced in Fig. S1, the relative noise $\Delta I_{signal}/2I_0$ detected in experiment remains the same as $\left(1/\sqrt{\beta} - \delta\right)$ is varied from null to 40 μrad. In particular, the observed variation is much larger than that estimated using Eq. (10). For instance, knowing the intensity fluctuation of 0.26%, $\Delta E_{s0}/\bar{E}_{s0}$ is found to be 0.13%.

Inserting $(1/\sqrt{\beta}-\delta)$~40 μrad into Eq. (10), the noise of $\Delta I_{signal}/2I_0$ is ~0.1 μrad, in contrast to 8 μrad shown in Fig. S1. Thus, the intensity fluctuation is not a major noise source for MOKE measurement.

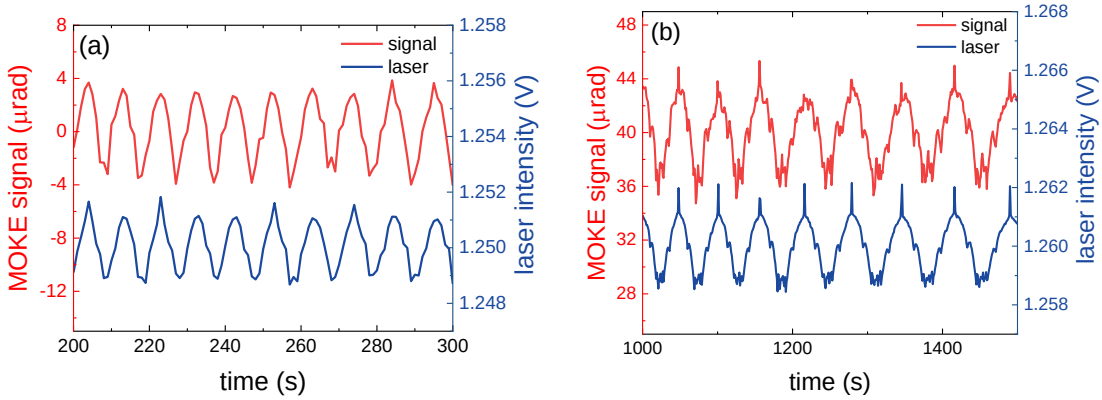

Fig. S1. (a) Fluctuations of $\Delta I_{signal}/2I_0$ and laser intensity when balancing the signal between two photodiodes in the detector. (b) The same as in (a) but setting $(1/\sqrt{\beta}-\delta)$~40 μrad.

The above discussion suggests polarization fluctuation must be one of the major noise sources. It is well-known that during a frequency-mode-sweeping process in HeNe laser cavity, the changes of *s*-mode and *p*-mode are out of phase that subject to the relation of $\Delta E_p/\bar{E}_p = -\Delta E_s/\bar{E}_s$[2]. Via fine adjustment of the waveplate angle, we may have $\delta = \bar{E}_{p0}/\bar{E}_{s0} = 1/\sqrt{\beta}$. Then, Eq. (9) turns into the following form

$$\frac{\Delta I_{signal}}{2I_0} \approx \frac{2}{\sqrt{\beta}}\frac{\Delta E_{s0}}{\bar{E}_{s0}} \tag{11}$$

Taking the values of $\beta = 1\times10^5$ and $\Delta E_{s0}/\bar{E}_{s0} = 0.13\%$, the relative change of the signal is found to be 8.2×10^{-6} rad, which agrees nicely with the experimental data.

3. Noise caused by laser pointing fluctuation

Lastly, we briefly address the MOKE noise induced by pointing stability of laser beam. We notice that the response of a detector may vary slightly over the detector surface. Therefore, a drift of laser pointing may cause imbalance between the two photoreceivers that were a priori in balance. To estimate the effect of laser pointing fluctuation, we used a piezo-controlled mirror to modulate the laser pointing direction and recorded the output voltage from the detector. Change of the laser position was monitored by a 4-quadrant detector concurrently. The results are given in Fig. S2, which demonstrates that variation of 25 μrad in laser pointing leads to approximately 2 μrad drift in MOKE measurement. Therefore, for high-accurate MOKE experiment beyond 1 μrad, one needs to stabilize the laser pointing noise down to a few μrad using, for instance, a beam-pointing

stabilizer. Thus, the selection of photodetectors with better uniformity will certainly help. Also, avoiding using tightly focused spots on the detectors is also recommended.

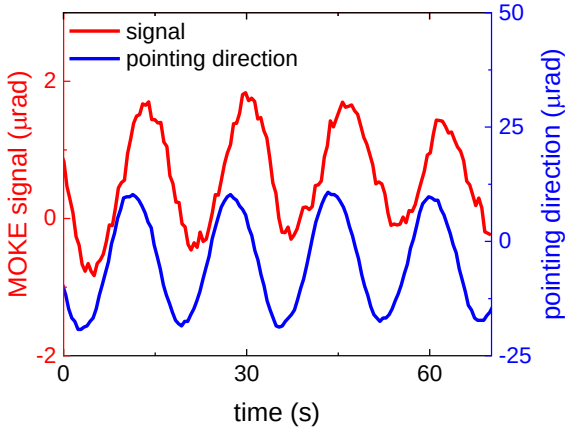

Fig. S2. Drift of MOKE signal versus the laser beam pointing angle.

Reference

1. S. Polisetty, J. Scheffler, S. Sahoo, Y. Wang, T. Mukherjee, X. He, and C. Binek, "Optimization of magneto-optical Kerr setup: Analyzing experimental assemblies using Jones matrix formalism," Review of Scientific Instruments **79,** 055107 (2008).
2. J. D. Ellis, K.-N. Joo, E. S. Buice, and J. W. Spronck, "Frequency stabilized three mode HeNe laser using nonlinear optical phenomena," Optics Express **18,** 1373 (2010).